A bricklayer lays an average of 150 bricks per hour. During the job he is called away and is replaced by a less experienced man, who averages 120 bricks per hour. The two men laid a total of 930 bricks in 7 hours. How long did each of the men work?

Last season two running backs on a football team ran for a combined total of 1550 yards. One ran 4 times as many yards as the other. How many yards were ran by each player?

A textbook has a total of 1382 pages which is broken into two parts. The second part of the book has 64 more pages than the first part. How many pages are in each part of the book.

Frank mowed his next door neighbor's lawn for a handful of dimes and nickels, 80 coins in all. Upon completing the job he counted out the coins and it came to $6.60. How many of each coin did he earn?

On Monday Joe bought 10 cups of coffee and 5 doughnuts for his office at the cost of $16.50. On Tuesday he bought 5 cups of coffee and 10 doughnuts for a total of $14.25. How much is a donut?

Crew A can assemble 2 cars in 5 days and crew B can assemble 3 cars in 7 days. If both crews together assembled 100 cars, with crew B working 10 days longer than crew A, how many days (rounded to the nearest day) must each crew have worked?

A tank can be filled by a pipe in 3 hours. The same tank can be emptied by another pipe in 4 hours. How much time would be needed to fill an empty tank if both pipes are on.

Two different gasolines are available, one containing 5% ethanol and the other 12% ethanol. How much of each (to the nearest litre) would be needed to fill 1000 litres of gas containing 10% ethanol?

A chemistry teacher needs to make 10 L of 42% sulphuric acid solution. The acid solutions available are 30% sulphuric acid and 50% sulphuric acid, by volume. How many liters of each solution must be mixed to make the 42% solution?

Person A and person B can lift 2240 lbs. combined at the gym. Person A can lift 90 more pounds than half of what person B can lift. How much can each person lift?

The perimeter of a rectangular garden is 62 feet. The length is 1 foot more than twice the width. Find the dimension of the garden.

A boat traveled 24 miles downstream in 2 hours. The return trip took twice as long. What is the speed of the boat in still water?

The sum of the digits of a two-digit number is 7. When the digits are reversed, the number is increased by 27. What is the number?

Find the partial fraction decomposition of the following equation: $$\frac{3x+1}{(x+1)(x-1)}$$

Find the partial fraction decomposition of the following equation: $$\frac{6x+18}{(x-1)(x+5)}$$

A bricklayer lays an average of 150 bricks per hour. During the job he is called away and is replaced by a less experienced man, who averages 120 bricks per hour. The two men laid a total of 930 bricks in 7 hours. How long did each of the men work?

let:

x = # of hours 1st bricklayer worked

y = # of hours 2nd bricklayer worked.

So:

$$x + y = 7 \qquad (1)$$

$$150x + 120y = 930 \qquad (2)$$

solve by substitution:

solve for x in equ. (1):

$$x + y = 7$$

$$\boxed{x = 7 - y} \qquad (1a)$$

sub. (1a) into (2):

$$150(7-y) + 120y = 930$$

$$1050 - 150y + 120y = 930$$

$$-30y = -120$$

$$\boxed{y = 4} \qquad (2a)$$

sub (2a) into (1a)

$$x = 7 - (4)$$

$$\boxed{x = 3}$$

$x = 3$ hours

$y = 4$ hours

Last season two running backs on a football team ran for a combined total of 1550 yards. One ran 4 times as many yards as the other. How many yards were ran by each player?

let:

x = # of yards running back A ran for.

y = # of yards running back B ran for:

So

$$x + y = 1550 \quad (1)$$

$$x = 4y \quad (2)$$

Solve by elimination:

$$x + y = 1550$$
$$-x + 4y = 0$$

$$5y = 1550$$

$$\boxed{y = 310}$$

Sub. $y = 310$ into (2)

$$x = (4)(310)$$

$$\boxed{x = 1240}$$

$$\boxed{\begin{aligned} x &= 1240 \text{ yards} \\ y &= 310 \text{ yards} \end{aligned}}$$

A textbook has a total of 1382 pages which is broken into two parts. The second part of the book has 64 more pages than the first part. How many pages are in each part of the book.

let:

x = # of pages in 1st section

y = # of pages in 2nd section

So:

$$x + y = 1382$$
$$y - x = 64$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1382 \\ 64 \end{bmatrix}$$

Solve using matrix row elimination:

$$\begin{bmatrix} 1 & 1 & 1382 \\ -1 & 1 & 64 \end{bmatrix}$$

$$R_2+R_1 \begin{bmatrix} 1 & 1 & 1382 \\ 0 & 2 & 1446 \end{bmatrix}$$

$$R_1-R_2^* \begin{bmatrix} 1 & 0 & 659 \\ 0 & 1 & 723 \end{bmatrix}$$

$$R_2/2 \begin{bmatrix} 1 & 0 & 659 \\ 0 & 1 & 723 \end{bmatrix}$$

$x = 659$ pages

$y = 723$ pages

Frank mowed his next door neighbor's lawn for a handful of dimes and nickels, 80 coins in all. Upon completing the job he counted out the coins and it came to $6.60. How many of each coin did he earn?

let:

x = # of dimes (worth 10¢ each)

y = # of nickels (worth 5¢ each)

so:

$$x + y = 80$$
$$10x + 5y = 660$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 10 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 80 \\ 660 \end{bmatrix}$$

solve using Cramer's rule:

$$x_i = \frac{\det(A_i)}{\det(A)}$$

$$x_1 = \frac{\det(A_1)}{\det(A)} = \frac{\det\begin{bmatrix} 80 & 1 \\ 660 & 5 \end{bmatrix}}{\det\begin{bmatrix} 1 & 1 \\ 10 & 5 \end{bmatrix}} = \frac{(80)(5) - (1)(660)}{(1)(5) - (1)(10)} = \frac{-260}{-5} = 52$$

$$x_2 = \frac{\det(A_2)}{\det(A)} = \frac{\det\begin{bmatrix} 1 & 80 \\ 10 & 660 \end{bmatrix}}{\det\begin{bmatrix} 1 & 1 \\ 10 & 5 \end{bmatrix}} = \frac{(1)(660) - (80)(10)}{(1)(5) - (1)(10)} = \frac{-140}{-5} = 28$$

$x = 52$ dimes

$y = 28$ nickels

On Monday Joe bought 10 cups of coffee and 5 doughnuts for his office at the cost of $16.50. On Tuesday he bought 5 cups of coffee and 10 doughnuts for a total of $14.25. How much is a donut?

let:

x = cost of coffee

y = cost of donut

so:

$$10x + 5y = 16.50 \quad (1)$$
$$5x + 10y = 14.25 \quad (2)$$

solve by substitution:

solve for x in equ. (2)

$$5x + 10y = 14.25$$
$$5x = 14.25 - 10y$$
$$\boxed{x = 2.85 - 2y} \quad (2a)$$

sub (2a) into (1):

$$10(2.85 - 2y) + 5y = 16.50$$
$$28.5 - 20y + 5y = 16.50$$
$$-15y = -12$$
$$\boxed{y = 0.80} \quad (1a)$$

sub (1a) into (2a)

$$x = 2.85 - 2(0.8)$$
$$\boxed{x = 1.25}$$

$$\boxed{\begin{aligned} x &= 1.25 \text{ \$} \\ y &= 0.80 \text{ \$} \end{aligned}}$$

Crew A can assemble 2 cars in 5 days and crew B can assemble 3 cars in 7 days. If both crews together assembled 100 cars, with crew B working 10 days longer than crew A, how many days (rounded to the nearest day) must each crew have worked?

let:

x = # of days crew A worked

y = # of days crew B worked.

So:

$$y - x = 10 \qquad (1)$$

$$\frac{2}{5}x + \frac{3}{7}y = 100 \qquad (2)$$

solve by elimination:

$$-\frac{2}{5}x + \frac{2}{5}y = \frac{20}{5}$$

$$\frac{2}{5}x + \frac{3}{7}y = 100$$

$$\frac{29}{35}y = 104$$

$$\boxed{y = \frac{3640}{29} \approx 126}$$

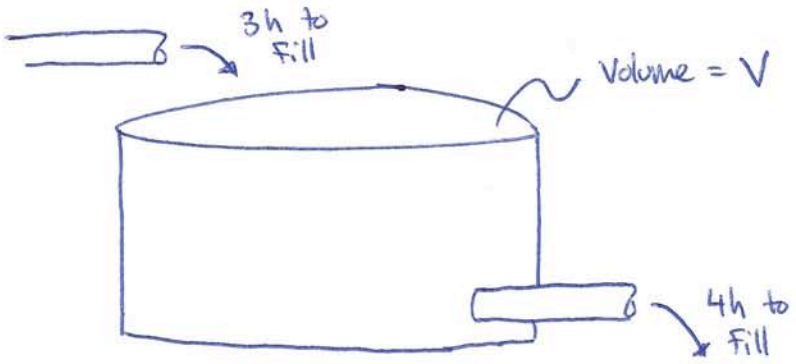
sub $y = 126$ into (1)

$$x = y - 10 = 126 - 10$$

$$\boxed{x = 116}$$

$\boxed{\begin{aligned} x &= 116 \text{ days} \\ y &= 126 \text{ days} \end{aligned}}$

A tank can be filled by a pipe in 3 hours. The same tank can be emptied by another pipe in 4 hours. How much time would be needed to fill an empty tank if both pipes are on.



let:

V = volume of tank

So:

$$\text{rate}_{\text{Filling}} = \frac{V}{t} = \frac{V}{3}$$

$$\text{rate}_{\text{empty}} = \frac{V}{t} = \frac{V}{4}$$

$$\text{rate}_{\text{total}} = \boxed{\frac{V}{3} - \frac{V}{4} = \frac{V}{t}}$$

solve for t:

$$\boxed{t = \frac{(4)(3)}{4-3} = 12 \text{ hours}}$$

Two different gasolines are available, one containing 5% ethanol and the other 12% ethanol. How much of each (to the nearest litre) would be needed to fill 1000 litres of gas containing 10% ethanol?

let:

x = # of litres of 5% ethanol

y = # of litres of 12% ethanol

so:

$$x + y = 1000 \quad (1)$$

$$0.05x + 0.12y = (0.10)(1000) \quad (2)$$

solve by substitution:

solve for x in eqn (1)

$$\boxed{x = 1000 - y} \quad (1a)$$

sub (1a) into (2)

$$(0.05)(1000 - y) + 0.12y = (0.10)(1000)$$

$$50 - 0.05y + 0.12y = 100$$

$$0.07y = 50$$

$$\boxed{y = 714} \quad (2a)$$

sub (2a) into (1a)

$$x = 1000 - (714)$$

$$\boxed{x = 286}$$

$\boxed{x = 286 \text{ litres}}$

$\boxed{y = 714 \text{ litres}}$

A chemistry teacher needs to make 10 L of 42% sulphuric acid solution. The acid solutions available are 30% sulphuric acid and 50% sulphuric acid, by volume. How many liters of each solution must be mixed to make the 42% solution?

let:

x = # of litres of 30% sulphuric acid

y = # of litres of 50% sulphuric acid

so:

$$x + y = 10 \qquad (1)$$

$$(0.30)x + (0.50)y = (0.42)(10) \qquad (2)$$

solve by elimination:

$$\begin{array}{rcl} x + y & = & 10 \\ -0.6x - y & = & -8.4 \\ \hline 0.4x & = & 1.6 \end{array}$$

$$\boxed{x = 4}$$

sub. $x = 4$ into eqn. (1)

$$(4) + y = 10$$

$$\boxed{y = 6}$$

$\boxed{\begin{array}{l} x = 4 \text{ litres} \\ y = 6 \text{ litres} \end{array}}$

Person A and person B can lift 2240 lbs. combined at the gym. Person A can lift 90 more pounds than half of what person B can lift. How much can each person lift?

let

x = pounds that person A can lift.

y = pounds that person B can lift.

so:

$$x + y = 2240 \qquad (1)$$

$$\frac{y}{2} + 90 = x \qquad (2)$$

solve by substitution:

sub (2) into (1):

$$\left(\frac{y}{2} + 90\right) + y = 2240$$

$$1.5y = 2150$$

$$\boxed{y \cong 1433}$$

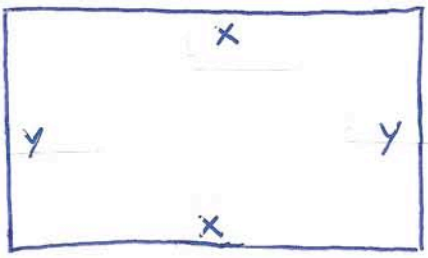
sub into (2)

$$x = \frac{(1433)}{2} + 90$$

$$\boxed{x \cong 807}$$

$$\boxed{\begin{aligned} x &= 807 \text{ lbs.} \\ y &= 1433 \text{ lbs.} \end{aligned}}$$

The perimeter of a rectangular garden is 62 feet. The length is 1 foot more than twice the width. Find the dimension of the garden.



let:

x = length of garden

y = width of garden

so

$$2x + 2y = 62 \quad (1)$$

$$2y + 1 = x \quad (2)$$

solve by substitution:

sub (2) into (1)

$$2(2y + 1) + 2y = 62$$

$$4y + 2 + 2y = 62$$

$$6y = 60$$

$$\boxed{y = 10}$$

sub back into (2)

$$x = 2(10) + 1$$

$$\boxed{x = 21}$$

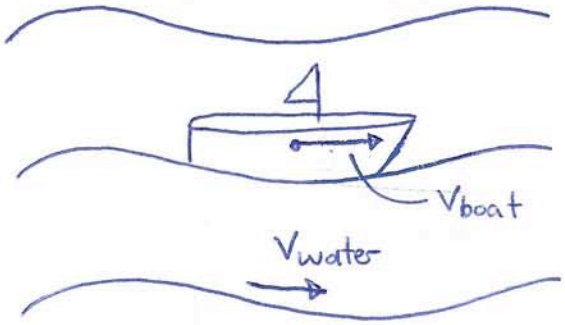
$\boxed{\begin{array}{l} x = 21 \text{ feet} \\ y = 10 \text{ feet} \end{array}}$

A boat traveled 24 miles downstream in 2 hours. The return trip took twice as long. What is the speed of the boat in still water?

let:

V_b = velocity of boat

V_w = velocity of water



so:

$$V_{total} = \frac{\text{distance}}{\text{time}}.$$

and

$$V_{downstream} = V_b + V_w = \frac{24}{2} = 12 \qquad (1)$$

$$V_{upstream} = V_b - V_w = \frac{24}{4} = 6 \qquad (2)$$

Solve by elimination:

$$V_b + V_w = 12$$

$$V_b - V_w = 6$$

$$2V_b = 18$$

$$\boxed{V_b = 9}$$

sub into eqn. (2)

$$9 - V_w = 6$$

$$V_w = 9 - 6$$

$$\boxed{V_w = 3}$$

$$\boxed{\begin{aligned} V_{boat} &= 9 \text{ miles/hour} \\ V_{water} &= 3 \text{ miles/hour} \end{aligned}}$$

The sum of the digits of a two-digit number is 7. When the digits are reversed, the number is increased by 27. What is the number?

let:

a = the 10's place holder number

b = the 1's place holder number

so

$$a + b = 7 \qquad (1)$$

$$(10b + a) = (10a + b) + 27 \qquad (2)$$

solve by substitution:

solve for a in equ. (1)

$$\boxed{a = 7 - b} \qquad (1a)$$

simplify equ. (2)

$$10b + a = 10a + b + 27$$

$$9a - 9b = -27 \qquad (2a)$$

sub equ. (1a) into (2a)

$$9(7 - b) - 9b = -27$$

$$63 - 9b - 9b = -27$$

$$-18b = -90$$

$$\boxed{b = 5}$$

sub into (1a):

$$a = 7 - (5)$$

$$\boxed{a = 2}$$

$$\boxed{\text{the number is } 25}$$

Find the partial fraction decomposition of the following equation: $\dfrac{3x+1}{(x+1)(x-1)}$

$$\frac{3x+1}{(x+1)(x-1)} = \frac{a}{x+1} + \frac{b}{x-1}$$

$$= \frac{(x-1)}{(x-1)}\frac{a}{(x+1)} + \frac{(x+1)}{(x+1)}\frac{b}{(x-1)}$$

$$= \frac{a(x-1) + b(x+1)}{(x+1)(x-1)}$$

$$\frac{3x+1}{(x+1)(x-1)} = \frac{ax - a + bx + b}{(x+1)(x-1)}$$

extract equations to solve:

x^1: $\quad 3 = a + b$

x^0: $\quad 1 = -a + b$

solve by elimination:

$$\begin{array}{r} 3 = a + b \\ 1 = -a + b \\ \hline 4 = 2b \end{array} \Rightarrow \boxed{b = 2}$$

sub into (2)

$$1 = -a + 2 \Rightarrow \boxed{a = 1}$$

$$\boxed{\frac{3x+1}{(x+1)(x-1)} = \frac{1}{x+1} + \frac{2}{x-1}}$$

Find the partial fraction decomposition of the following equation:

$$\frac{6x+18}{(x-1)(x+5)}$$

$$\frac{6x+18}{(x-1)(x+5)} = \frac{a}{(x-1)} + \frac{b}{(x+5)}$$

$$= \frac{a(x+5) + b(x-1)}{(x-1)(x+5)}$$

$$\frac{6x+18}{(x-1)(x+5)} = \frac{ax + 5a + bx - b}{(x-1)(x+5)}$$

extract equations to solve:

x^1: $\quad 6 = a + b \qquad (1)$

x^0: $\quad 18 = 5a - b \qquad (2)$

solve by elimination:

$$\begin{array}{r} 6 = a + b \\ 18 = 5a - b \\ \hline 24 = 6a \end{array} \quad \Rightarrow \boxed{a = 4}$$

sub into (1)

$$6 = 4 + b \quad \Rightarrow \boxed{b = 2}$$

$$\boxed{\frac{6x+18}{(x-1)(x+5)} = \frac{4}{x-1} + \frac{2}{x+5}}$$