Instructor:	Frank Secretain
Course:	Math 1004
Date:	December 13, 2024
Assessment:	Test 4 a
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	5 questions worth 20 marks
Percentage of final grade:	20% of final grade

Formula Sheet

Order of Operations

$$ac + bc = c(a + b)$$

exponents

$$a^n a^m = a^{n+m}$$

$$(a^n)^m = a^{nm}$$

$$(ab)^n = a^n b^n$$

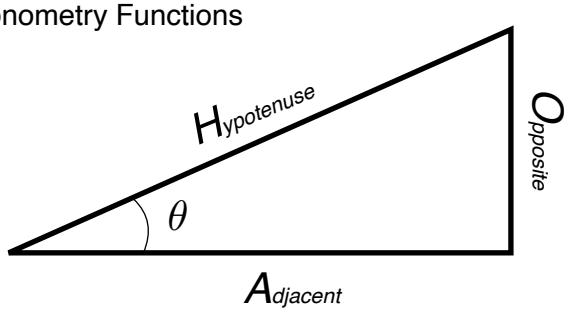
$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

radicals

$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

Trigonometry Functions



$$sin(\theta) = \frac{O}{H} \qquad sin^{-1}\left(\frac{O}{H}\right) = \theta$$

$$cos(\theta) = \frac{A}{H} \qquad cos^{-1}\left(\frac{A}{H}\right) = \theta$$

$$tan(\theta) = \frac{O}{A} \qquad tan^{-1}\left(\frac{O}{A}\right) = \theta$$

Pythagoras Theorem

$$H^2 = O^2 + A^2$$

Relative Velocity

$$\vec{v}_{\frac{A}{C}} = \vec{v}_{\frac{A}{B}} + \vec{v}_{\frac{B}{C}}$$

$$\vec{v}_{\frac{B}{A}} = -\vec{v}_{\frac{A}{B}}$$

Linear equations (Cramer's rule)

$$x_i = \frac{det(A_i)}{det(A)}$$

Forms of a 1st order polynomial

$$y = ax + b$$

Forms of a 2nd order polynomial

$$y = ax^2 + bx + c$$

$$y = a(x - h)^2 + k$$

$$y = (x - m)(x - n)$$

Quadratic Equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Unit Conversions

angles

$$2\pi = 6.28 \text{ rad} = 360^o$$

mass

$$1 \text{ kg} = 2.2 \text{ lbs.}$$

lengths

$$1 \text{ mile} = 1.6 \text{ km}$$

$$1 \text{ inch} = 2.54 \text{ cm}$$

$$1 \text{ m} = 3.3 \text{ ft}$$

volumes

$$1 \text{ gallon} = 3.78 \text{ Litres}$$

(2 marks) Solve the each expression and keep the correct number of significant digits.

(380)(3.2) + 120

(2.43)(3991.76)+300

(2 marks) Given the standard unit conversion table on the formula sheet (1st page), convert each of the numbers to the stated units.

$$3.5\ \frac{\text{radians}}{\text{s}} \rightarrow \frac{\text{degrees}}{\text{minute}}$$

$$873\ \frac{\text{miles}^2}{\text{hour}} \rightarrow \frac{\text{feet}^2}{\text{second}}$$

(4 marks) You run 150 m West, 120 m at 30^0 North of East and 70 m at 60^0 West of North. How far are you from where you started?

(2 marks each) Solve for x in the following equations.

$$IR_o + Ix - 5 = V_o$$

$$\frac{bx + c}{2} - 3b = c$$

$$\frac{a(x-1)+2}{x-1}+b=3$$

$$\frac{x+1}{x}+3=\frac{\rho\sin(\theta)}{2}$$

(4 marks) Solve for x and y in the following equation.

$$\frac{3(x+y)-1}{x+y}+1=5$$

$$\frac{x+1}{x+y}+1=9$$

(2 marks) Solve the each expression and keep the correct number of significant digits.

(380)(3.2) + 120

$1216 + 120 = 1336$

$= 1300$

(2.43)(3991.76)+300

$9699.9768 + 300 = 9999.9768$

$= 10000$

$= 1.00 \times 10^4$

(2 marks) Given the standard unit conversion table on the formula sheet (1st page), convert each of the numbers to the stated units.

$$3.5 \frac{\text{radians}}{\text{s}} \rightarrow \frac{\text{degrees}}{\text{minute}}$$

$3.5 \frac{\text{radians}}{\text{sec}} \left(\frac{360 \text{ degrees}}{2\pi \text{ radians}}\right)\left(\frac{60 \text{ sec}}{1 \text{ min}}\right) = 12032 \frac{\text{deg}}{\text{sec}}$

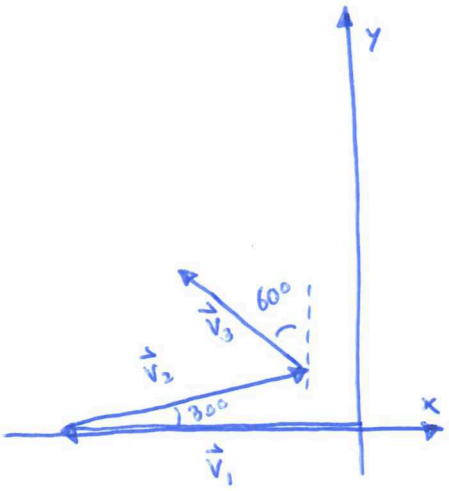
$= 12000 \frac{\text{deg}}{\text{sec}}$

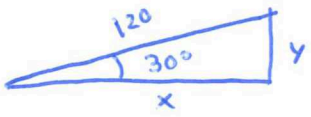
$$873 \frac{\text{miles}^2}{\text{hour}} \rightarrow \frac{\text{feet}^2}{\text{second}}$$

$873 \frac{\text{miles}^2}{\text{hour}} \left(\frac{1.6^2 \text{ km}^2}{1^2 \text{ mile}^2}\right)\left(\frac{1000^2 \text{ m}^2}{1^2 \text{ km}^2}\right)\left(\frac{3.3^2 \text{ ft}^2}{1^2 \text{ m}^2}\right)\left(\frac{1 \text{ hour}}{60 \text{ min}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right)$

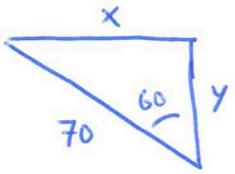
$6760512 \frac{\text{feet}^2}{\text{second}} = 6\,760\,000 \frac{\text{feet}^2}{\text{sec}}$

(4 marks) You run 150 m West, 120 m at 30° North of East and 70 m at 60° West of North. How far are you from where you started?





$x = 120 \cos(30)$
$= 103.92$
$y = 120 \sin(30)$
$= 60$



$x = 70 \sin(60)$
$= 60.62$
$y = 70 \cos(60)$
$= 35$

$\vec{V}_1 = -150\,\hat{x} + 0\,\hat{y}$

$\vec{V}_2 = 103.92\,\hat{x} + 60\,\hat{y}$

$\vec{V}_3 = -60.62\,\hat{x} + 35\,\hat{y}$

$\vec{V}_f = -106.7\,\hat{x} + 95\,\hat{y}$

$|\vec{V}_f| = \sqrt{(-106.7) + (95)}$

$= 142.86$

$\boxed{|\vec{V}_f| = 140 \text{ m}}$

(2 marks each) Solve for x in the following equations.

$$IR_o + Ix - 5 = V_o$$

$$Ix = V_o - IR_o + 5$$

$$x = \frac{V_o - IR_o + 5}{I}$$

$$\frac{bx + c}{2} - 3b = c$$

$$bx + c = 2(c + 3b)$$

$$x = \frac{2(c + 3b) - c}{b} = \frac{c + 6b}{b}$$

$$\frac{a(x-1)+2}{x-1}+b=3$$

$a(x-1)+2+b(x-1)=3(x-1)$

$ax-a+2+bx-b=3x-3$

$ax+bx-3x=a+b-5$

$$\boxed{x=\frac{a+b-5}{a+b-3}}$$

$$\frac{x+1}{x}+3=\frac{\rho\sin(\theta)}{2}$$

$2(x+1)+6x=\rho x\sin(\theta)$

$2x+2+6x=\rho x\sin(\theta)$

$8x-\rho x\sin(\theta)=-2$

$$\boxed{x=\frac{-2}{8-\rho\sin\theta}=\frac{2}{\rho\sin\theta-8}}$$

(4 marks) Solve for x and y in the following equation.

$$\frac{3(x+y)-1}{x+y}+1=5 \qquad (1)$$

$$\frac{x+1}{x+y}+1=9 \qquad (2)$$

Simplify (1):

$3x + 3y - 1 = 4x + 4y$

$x + y = -1$ (1a)

Simplify (2):

$x + 1 = 8x + 8y$

$7x + 8y = 1$ (2a)

Solve for x in (1a)

$x = -1 - y$ (1b)

sub. (1b) into (2a)

$7[-1-y] + 8y = 1$

$-7 - 7y + 8y = 1$

$\boxed{y = 8}$ (2b)

sub (2b) into (1a)

$x = -1 - [8]$

$\boxed{x = -9}$

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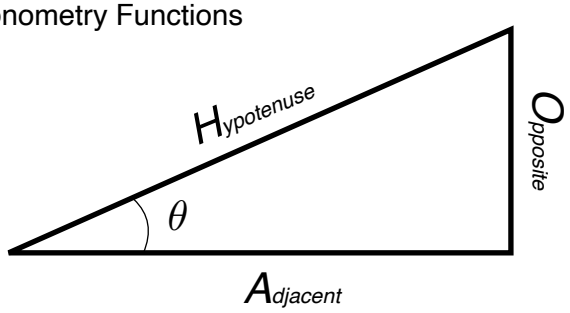

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(2 marks) Solve the each expression and keep the correct number of significant digits.

(480)(3.2) + 120

(2.43)(3991.76)+10300

(2 marks) Given the standard unit conversion table on the formula sheet (1st page), convert each of the numbers to the stated units.

$$5.9\ \frac{radians}{seconds} \rightarrow \frac{degrees}{minute}$$

$$0.54\ \frac{miles^2}{hour} \rightarrow \frac{feet^2}{second}$$

(4 marks) You run 190 m West, 120 m at 50^0 North of East and 70 m at 30^0 West of North. How far are you from where you started?

(2 marks each) Solve for x in the following equations.

$$v^2 x - 4 + \Gamma = V_o$$

$$\frac{ax - h}{3} - 5b = c$$

$$\frac{b(x-1)+3}{x-1}+b=5$$

$$\frac{x}{x-1}+5=\frac{b\sin(\phi)}{4}$$

(4 marks) Solve for x and y in the following equation.

$$\frac{7(x-y)-1}{x-y}+1=5$$

$$\frac{x+1}{x+y}+1=9$$

(2 marks) Solve the each expression and keep the correct number of significant digits.

(480)(3.2) + 120

$1536 + 120 = 1656$ (sig figs: 2, 2, -1, -2)

$= \boxed{1700}$

(2.43)(3991.76)+10300

$9699.9768 + 10300 = 19999.9768$ (sig figs: 3, 6, -1, -2)

$= 20000 = \boxed{2.00 \times 10^4}$ (sig figs: -2)

(2 marks) Given the standard unit conversion table on the formula sheet (1st page), convert each of the numbers to the stated units.

$$5.9 \ \frac{radians}{seconds} \rightarrow \frac{degrees}{minute}$$

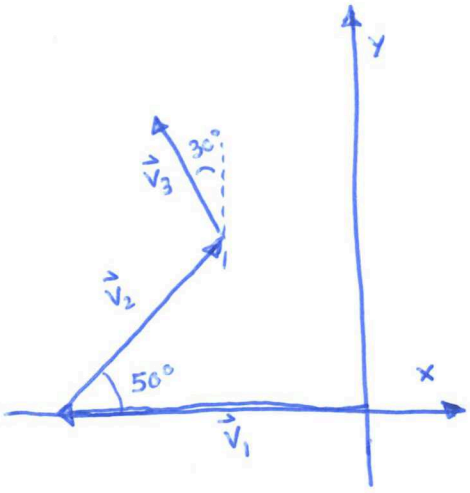
$$5.9 \ \frac{rads}{s}\left(\frac{360 \ deg}{2\pi \ rads}\right)\left(\frac{60 \ s}{1 \ min}\right) = \boxed{20283 \ \frac{deg}{min}} = \boxed{2.0 \times 10^4 \ \frac{deg}{min}}$$

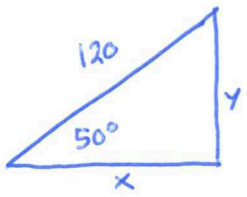
$$0.54 \ \frac{miles^2}{hour} \rightarrow \frac{feet^2}{second}$$

$$0.54 \ \frac{miles^2}{hour}\left(\frac{1.6^2 \ km^2}{1^2 \ mile^2}\right)\left(\frac{1000^2 \ m^2}{1^2 \ km^2}\right)\left(\frac{3.3^2 \ ft^2}{1^2 \ m^2}\right)\left(\frac{1 \ hour}{60 \ min}\right)\left(\frac{1 \ min}{60 \ sec}\right)$$

$$\boxed{4182 \ \frac{ft^2}{sec} = 4.2 \times 10^3 \ \frac{ft^2}{sec}}$$

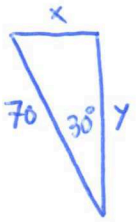
(4 marks) You run 190 m West, 120 m at 50⁰ North of East and 70 m at 30⁰ West of North. How far are you from where you started?





$x = 120\cos(50) = 77.13$

$y = 120\sin(50) = 91.93$



$x = 70\sin(30) = 35$

$y = 70\cos(30) = 60.62$

$\vec{V}_1 = -190\,\hat{x} + 0\,\hat{y}$

$\vec{V}_2 = 77.13\,\hat{x} + 91.93\,\hat{y}$

$+\ \vec{V}_3 = -35\,\hat{x} + 60.62\,\hat{y}$

$\vec{V}_f = -147.87\,\hat{x} + 152.55\,\hat{y}$

$$|\vec{V}_f| = \sqrt{(-147.87)^2 + (152.55)^2}$$

$$= 212.45$$

$$\boxed{|\vec{V}_f| = 210 \text{ m}}$$

(2 marks each) Solve for x in the following equations.

$$v^2x - 4 + \Gamma = V_o$$

$+4 \quad -\Gamma \qquad +4 \quad -\Gamma$

$$\frac{v^2x}{v^2} = \frac{V_o - \Gamma + 4}{v^2}$$

$$\boxed{x = \frac{V_o - \Gamma + 4}{v^2}}$$

$$\frac{ax - h}{3} - 5b = c$$

$$ax - h = 3(c + 5b)$$

$$\boxed{x = \frac{3(c + 5b) + h}{a}}$$

$$\frac{b(x-1)+3}{x-1}+b=5$$

$b(x-1)+3 + b(x-1) = 5(x-1)$

$bx - b + 3 + bx - b = 5x - 5$

$2bx - 2b + 3 = 5x - 5$

$2bx - 5x = 2b - 8$

$$\boxed{x = \frac{2b-8}{2b-5}}$$

$$\frac{x}{x-1}+5=\frac{b\sin(\phi)}{4}$$

$4x + 20(x-1) = b\sin(\phi)(x-1)$

$4x + 20x - 20 = bx\sin(\phi) - b\sin(\phi)$

$24x - bx\sin(\phi) = 20 - b\sin(\phi)$

$$\boxed{x = \frac{20 - b\sin(\phi)}{24 - b\sin(\phi)}}$$

(4 marks) Solve for x and y in the following equation.

$$\frac{7(x-y)-1}{x-y}+1=5 \qquad (1)$$

$$\frac{x+1}{x+y}+1=9 \qquad (2)$$

Simplify (1)

$$7x-7y-1=4x-4y$$

$$3x-3y=1 \qquad (1a)$$

Simplify (2)

$$x+1=8x+8y$$

$$7x+8y=1 \qquad (2a)$$

Solve for x in (1a):

$$x=\frac{1+3y}{3} \qquad (1b)$$

sub (1b) into (2a)

$$7\left[\frac{1+3y}{3}\right]+8y=1$$

$$7+21y+24y=3$$

$$45y=-4$$

$$\boxed{y=-\frac{4}{45}=-0.0\overline{8}} \qquad (2b)$$

Sub (2b) into (1b)

$$x=\frac{1+3\left[-\frac{4}{45}\right]}{3}$$

$$\boxed{x=\frac{11}{45}=0.2\overline{4}}$$