Instructor:	Frank Secretain
Course:	Math 1004
Date:	November 29, 2024
Assessment:	Test 3 a
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	3 questions worth 20 marks
Percentage of final grade:	20% of final grade

Formula Sheet

Order of Operations

$$ac + bc = c(a + b)$$

exponents

$$a^n a^m = a^{n+m}$$

$$(a^n)^m = a^{nm}$$

$$(ab)^n = a^n b^n$$

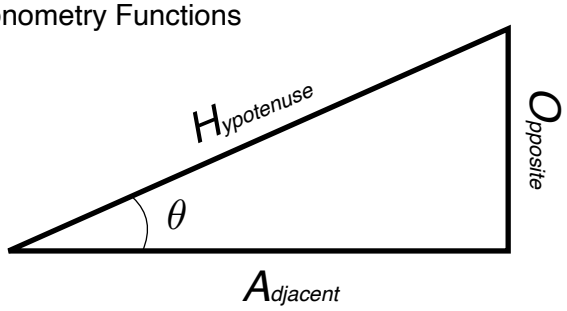
$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

radicals

$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

Trigonometry Functions



$$sin(\theta) = \frac{O}{H} \qquad sin^{-1}\left(\frac{O}{H}\right) = \theta$$

$$cos(\theta) = \frac{A}{H} \qquad cos^{-1}\left(\frac{A}{H}\right) = \theta$$

$$tan(\theta) = \frac{O}{A} \qquad tan^{-1}\left(\frac{O}{A}\right) = \theta$$

Pythagoras Theorem

$$H^2 = O^2 + A^2$$

Relative Velocity

$$\vec{v}_{\frac{A}{C}} = \vec{v}_{\frac{A}{B}} + \vec{v}_{\frac{B}{C}}$$

$$\vec{v}_{\frac{B}{A}} = -\vec{v}_{\frac{A}{B}}$$

Linear equations (Cramer's rule)

$$x_i = \frac{det(A_i)}{det(A)}$$

Forms of a 1st order polynomial

$$y = ax + b$$

Forms of a 2nd order polynomial

$$y = ax^2 + bx + c$$

$$y = a(x - h)^2 + k$$

$$y = (x - m)(x - n)$$

Quadratic Equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Unit Conversions

angles

$$2\pi = 6.28\,\text{rad} = 360^o$$

mass

$$1\,\text{kg} = 2.2\,\text{lbs.}$$

lengths

$$1\,\text{mile} = 1.6\,\text{km}$$

$$1\,\text{inch} = 2.54\,\text{cm}$$

$$1\,\text{m} = 3.3\,\text{ft}$$

volumes

$$1\,\text{gallon} = 3.78\,\text{Litres}$$

.(3 marks each) Solve for x in the following equations.

$$3(x+1) - b = 4x + 2$$

$$\frac{4b\sin(\theta) - x}{2} + b = \alpha^2$$

$$\frac{x-1}{x+1} + x = 2$$

$$\frac{x-1}{x+1} = a$$

(4 marks) Solve for E and I in the following equations.

$$E = IR$$

$$P = IE$$

(4 marks) 8 buckets and 13 cups can hold 80 litres of water. However, 7 buckets minus 5 cups of water holds 10 litres of water. What is the volume of the bucket and cup.

.(3 marks each) Solve for x in the following equations.

$$3(x+1)-b=4x+2$$

$3x + 3 - b = 4x + 2$

$-x = b - 1$

$\boxed{x = 1 - b}$

$$\frac{4b\sin(\theta)-x}{2}+b=\alpha^2$$

$\frac{4b\sin\theta - x}{2} = \alpha^2 - b$

$4b\sin\theta - x = 2(\alpha^2 - b)$

$-x = 2(\alpha^2 - b) - 4b\sin\theta$

$\boxed{x = 4b\sin\theta - 2(\alpha^2 - b)}$

$$\frac{x-1}{x+1}+x=2$$

$x-1+x(x+1)=2(x+1)$

$x-1+x^2+x=2x+2$

$x^2=3$

$$\boxed{x=\pm\sqrt{3}}$$

$$\frac{x-1}{x+1}=a$$

$x-1=a(x+1)$

$x-1=ax+a$

$x-ax=1+a$

$x(1-a)=1+a$

$$\boxed{x=\frac{1+a}{1-a}}$$

(4 marks) Solve for E and I in the following equations.

$$E = IR \qquad (1)$$

$$P = IE \qquad (2)$$

sub (1) into (2)

$$P = I[IR]$$

$$P = I^2 R$$

$$I^2 = \frac{P}{R}$$

$$\boxed{I = \sqrt{\frac{P}{R}}} \qquad (2a)$$

sub (2a) into (1)

$$E = \left[\sqrt{\frac{P}{R}}\right] R$$

$$\boxed{E = R\sqrt{\frac{P}{R}} = \sqrt{R^2}\sqrt{\frac{P}{R}} = \sqrt{\frac{R^2 P}{R}} = \sqrt{RP}}$$

(4 marks) 8 buckets and 13 cups can hold 80 litres of water. However, 7 buckets minus 5 cups of water holds 10 litres of water. What is the volume of the bucket and cup.

let b = volume of a bucket

c = " " " cup

so

$$8b + 13c = 80 \quad (1)$$

$$7b - 5c = 10 \quad (2)$$

solve for b in (2):

$$b = \frac{10 + 5c}{7} \quad (2a)$$

sub (2a) into (1):

$$8\left[\frac{10 + 5c}{7}\right] + 13c = 80$$

$$8(10 + 5c) + 91c = 560$$

$$80 + 40c + 91c = 560$$

$$131c = 480$$

$$c = \frac{480}{131} = 3.66 \text{ litres} \quad (1a)$$

sub (1a) into (2a)

$$b = \frac{10 + 5[3.66]}{7} = 4.05 \text{ litres}$$

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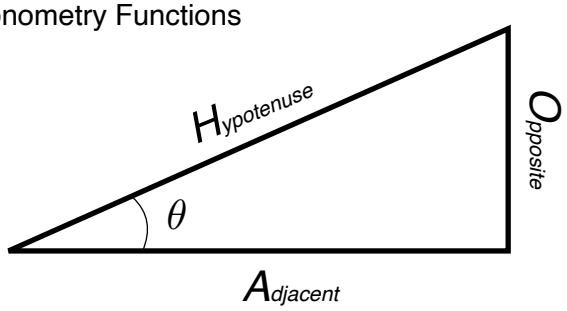

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.(3 marks each) Solve for x in the following equations.

$$5(x-1) = b - 5x + 1$$

$$b + \frac{x - 4\cos(\theta)}{3} = \alpha^2$$

$$\frac{x-1}{x+1} + x = 2$$

$$\frac{x-1}{x+1} = a$$

(4 marks) Solve for E and I in the following equations.

$$E = IR$$

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(4 marks) 8 buckets and 10 cups can hold 80 litres of water. However, 7 buckets minus 5 cups of water holds 10 litres of water. What is the volume of the bucket and cup.

.(3 marks each) Solve for x in the following equations.

$$5(x-1) = b - 5x + 1$$

$5x - 5 = b - 5x + 1$

$10x = b + 6$

$$\boxed{x = \frac{b+6}{10}}$$

$$b + \frac{x - 4\cos(\theta)}{3} = \alpha^2$$

$$\frac{x - 4\cos\theta}{3} = \alpha^2 - b$$

$x - 4\cos\theta = 3(\alpha^2 - b)$

$$\boxed{x = 3(\alpha^2 - b) + 4\cos\theta}$$

$$\frac{x-1}{x+1} + x = 2$$

$x - 1 + x(x+1) = 2(x+1)$

$x - 1 + x^2 + x = 2x + 2$

$x^2 = 3$

$$\boxed{x = \pm\sqrt{3}}$$

$$\frac{x-1}{x+1} = a$$

$x - 1 = a(x+1)$

$x - 1 = ax + a$

$x - ax = 1 + a$

$x(1-a) = 1 + a$

$$\boxed{x = \frac{1+a}{1-a}}$$

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c = " " " cup

so

$$8b + 10c = 80 \quad (1)$$

$$7b - 5c = 10 \quad (2)$$

solve for b c in (2)

$$c = \frac{7b - 10}{5} \quad (2a)$$

sub (2a) into (1):

$$8b + \cancel{10}\left[\frac{7b - 10}{\cancel{5}}\right] = 80$$

$$8b + 2(7b - 10) = 80$$

$$8b + 14b - 20 = 80$$

$$22b = 100$$

$$\boxed{b = \frac{100}{22} = \frac{50}{11} = 4.54} \quad (1a)$$

sub (1a) into (2a)

$$\boxed{c = \frac{7[4.54] - 10}{5} = \frac{48}{11} = 4.36}$$