Instructor:	Frank Secretain
Course:	Math 101
Assessment:	Test 1
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	6 questions worth 30 marks
Percentage of final grade:	23% of final grade

Formula Sheet

Order of Operations

$$ac + bc = c(a + b)$$

exponents

$$a^n a^m = a^{n+m}$$

$$(a^n)^m = a^{nm}$$

$$(ab)^n = a^n b^n$$

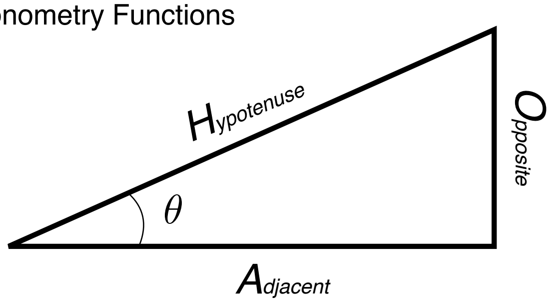
$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

radicals

$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

Trigonometry Functions



$$sin(\theta) = \frac{O}{H} \qquad sin^{-1}\left(\frac{O}{H}\right) = \theta$$

$$cos(\theta) = \frac{A}{H} \qquad cos^{-1}\left(\frac{A}{H}\right) = \theta$$

$$tan(\theta) = \frac{O}{A} \qquad tan^{-1}\left(\frac{O}{A}\right) = \theta$$

Pythagoras Theorem

$$H^2 = O^2 + A^2$$

Relative Velocity

$$\vec{v}_{\frac{A}{C}} = \vec{v}_{\frac{A}{B}} + \vec{v}_{\frac{B}{C}}$$

Linear equations (Cramer's rule)

$$x_i = \frac{det(A_i)}{det(A)}$$

Forms of a 2nd order polynomial

$$y = ax^2 + bx + c$$

$$y = a(x - h)^2 + k$$

$$y = (x - m)(x - n)$$

Unit Conversions

angles

$$2\pi = 6.28 \text{ rad} = 360^o$$

mass

$$1 \text{ kg} = 2.2 \text{ lbs.}$$

lengths

$$1 \text{ mile} = 1.6 \text{ km}$$

$$1 \text{ inch} = 2.54 \text{ cm}$$

$$1 \text{ m} = 3.3 \text{ ft}$$

volumes

$$1 \text{ gallon} = 3.78 \text{ Litres}$$

(4 marks) Match the "type of number" with the best "example number". Draw a line to match the "type of number" to the "example number" to indicate your answer.

whole	$\sqrt{-2}$
integer	$\sqrt{9}$
imaginary	$-\sqrt{9}$
irrational	$\sqrt{2}$

(3 marks) Solve the each expression and keep the correct number of significant digits.

120+23.853

23.82+(0.012)(4802.3)

4655.3+(1.30)(4111.38)

(2 marks) Convert each number into scientific notation.

340.0

0.0000100

(3 marks) Convert each of the numbers to the stated units.

$$93^o \rightarrow \text{radians}$$

$$87\ \frac{\text{miles}}{\text{hour}} \rightarrow \frac{\text{m}}{\text{s}}$$

$$2.4\ \frac{\text{gallons}}{\text{inch}^3} \rightarrow \frac{\text{L}}{\text{mm}^3}$$

(5 marks) You run 50 m North, 20 m East, 40 m at 25^0 South of East and an “unknown distance and direction”. Your final position relative to your starting point is 10 m 10^0 East of North. How far did you run for the “unknown distance”?

(13 marks) Solve for x in the following equations

$$4x + 2(x - 1) = 7$$

$$\frac{2 + 5(x - 1)}{x - 1} = 15 - 3$$

$$\frac{7 - 2(2x - 3 + x) - 1}{x + 1} + 4 = 9$$

$$\alpha x + \beta(x - \delta) = \theta$$

$$\frac{\eta + \rho(x - \sigma)}{x - \phi} + \omega = \tau$$

$$-m\omega^2 R = x\cos(\theta) - \left[\frac{1}{\cos(\theta)}(mg - x\sin(\theta))\right]\sin(\theta)$$

(4 marks) Match the "type of number" with the best "example number". Draw a line to match the "type of number" to the "example number" to indicate your answer.

whole	$\sqrt{-2} = \sqrt{(-1)(2)} = \sqrt{-1}\sqrt{2} = \sqrt{2}\,i$
integer	$\sqrt{9} = 3$
imaginary	$-\sqrt{9} = -3$
irrational	$\sqrt{2} = \sqrt{2}$

(3 marks) Solve the each expression and keep the correct number of significant digits.

$= 120+23.853$

$= 143.853$

$= 140$

$= 23.82+(0.012)(4802.3)$

$= 23.82 + 57.6276$

$= 81.4476$

$= 81$

$= 4655.3+(1.30)(4111.38)$

$= 4655.3 + 5344.794$

$= 10000.094$

$= 1.000 \times 10^4$

(2 marks) Convert each number into scientific notation.

340.0 $= 3.400 \times 10^{2}$

0.0000100 $= 1.00 \times 10^{-5}$

(3 marks) Convert each of the numbers to the stated units.

$93^{o} \rightarrow \text{radians}$

$$93^{\circ} \left(\frac{2\pi}{360^{\circ}}\right) = \frac{31\pi}{60} = 0.51\bar{6}\pi = 1.623 \text{ rads}$$

$87 \ \frac{\text{miles}}{\text{hour}} \rightarrow \frac{\text{m}}{\text{s}}$

$$87 \ \frac{\text{miles}}{\text{hour}} \left(\frac{1.6 \text{ km}}{1 \text{ mile}}\right)\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ hour}}{60 \text{ min}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right)$$

$$= 38.67 \ \frac{\text{m}}{\text{s}}$$

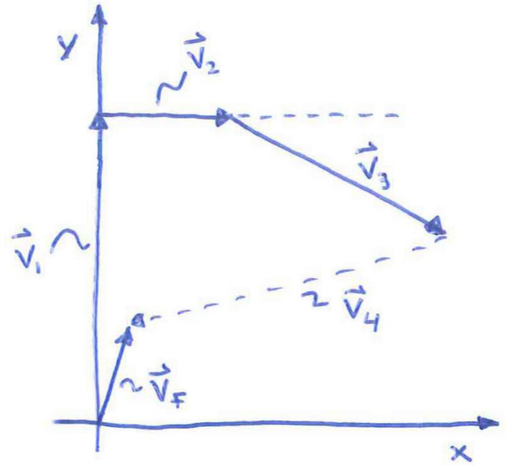
$2.4 \ \frac{\text{gallons}}{\text{inch}^3} \rightarrow \frac{\text{L}}{\text{mm}^3}$

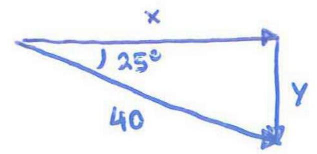
$$2.4 \ \frac{\text{gallons}}{\text{inch}^3} \left(\frac{3.78 \text{ L}}{1 \text{ gallon}}\right)\left(\frac{1 \text{ inch}}{2.54 \text{ cm}}\right)^3\left(\frac{1 \text{ cm}}{10 \text{ mm}}\right)^3$$

$$= 0.000554 \ \frac{\text{L}}{\text{mm}^3}$$

$$= 5.54 \times 10^{-4} \ \frac{\text{L}}{\text{mm}^3}$$

(5 marks) You run 50 m North, 20 m East, 40 m at 25º South of East and an "unknown distance and direction". Your final position relative to your starting point is 10 m 10º East of North. How far did you run for the "unknown distance"?





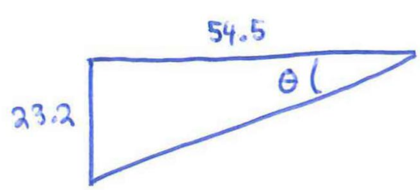
$x = 40 \cos 25$
$= 36.3$
$y = 40 \sin 25$
$= 16.9$

$x = 10 \sin 10$
$= 1.74$
$y = 10 \cos 10$
$= 9.85$

$$\vec{V}_1 + \vec{V}_2 + \vec{V}_3 + \vec{V}_4 = \vec{V}_F$$

$$\boxed{\vec{V}_4 = \vec{V}_F - \vec{V}_1 - \vec{V}_2 - \vec{V}_3}$$

$$\begin{aligned} V_F &= 1.74\,\hat{x} + 9.85\,\hat{y} \\ -\vec{V}_1 &= 0\,\hat{x} - 50\,\hat{y} \\ -\vec{V}_2 &= -20\,\hat{x} + 0\,\hat{y} \\ -\vec{V}_3 &= -36.3\,\hat{x} + 16.9\,\hat{y} \\ \hline \vec{V}_4 &= -54.5\,\hat{x} - 23.2\,\hat{y} \end{aligned}$$



$$|\vec{V}_4| = \sqrt{(-54.5)^2 + (-23.2)^2}$$

$$\boxed{|\vec{V}_4| = 59.3\text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{23.2}{54.5}\right) =$$

$$\boxed{\theta = 23°}$$

(13 marks) Solve for x in the following equations

$$4x + 2(x - 1) = 7$$

$4x + 2x - 2 = 7$

$6x = 9$

$x = \frac{9}{6}$

$x = \frac{3}{2} = 1.5$

$$\frac{2 + 5(x - 1)}{x - 1} = 15 - 3$$

$\frac{2 + 5x - 5}{x - 1} = 12$

$5x - 3 = 12x - 12$

$-7x = -9$

$x = \frac{9}{7}$

$x = \frac{9}{7} \cong 1.29$

$$\frac{7-2(2x-3+x)-1}{x+1}+4=9$$

$$\frac{6-2(3x-3)}{x+1}=5$$

$$6-6x+6=5x+5$$

$$-11x=-7$$

$$x=\frac{7}{11}$$

$$\boxed{x=\frac{7}{11}=0.\overline{63}}$$

$$\alpha x+\beta(x-\delta)=\theta$$

$$\alpha x+\beta x-\beta\delta=\theta$$

$$x(\alpha+\beta)=\theta+\beta\delta$$

$$x=\frac{\theta+\beta\delta}{\alpha+\beta}$$

$$\boxed{x=\frac{\theta+\beta\delta}{\alpha+\beta}}$$

$$\frac{\eta + \rho(x-\sigma)}{x-\phi} + \omega = \tau$$

$$\eta + \rho x - \rho\sigma + \omega(x-\phi) = \tau(x-\phi)$$

$$\eta + \rho x - \rho\sigma + \omega x - \phi\omega = \tau x - \tau\phi$$

$$\rho x + \omega x - \tau x = \phi\omega - \tau\phi - \eta + \rho\sigma$$

$$x(\rho + \omega - \tau) = \phi\omega - \tau\phi - \eta + \rho\sigma$$

$$\boxed{x = \frac{\phi\omega - \tau\phi - \eta + \rho\sigma}{\rho + \omega - \tau}}$$

$$-m\omega^2 R = x\cos(\theta) - \left[\frac{1}{\cos(\theta)}(mg - x\sin(\theta))\right]\sin(\theta)$$

$$-m\omega^2 R = x\cos\theta - \frac{\sin\theta}{\cos\theta} mg + \frac{\sin^2\theta}{\cos\theta} x$$

$$x\cos\theta + x\frac{\sin^2\theta}{\cos\theta} = \frac{\sin\theta}{\cos\theta} mg - m\omega^2 R$$

$$x = \frac{\frac{\sin\theta}{\cos\theta} mg - m\omega^2 R}{\cos\theta + \frac{\sin^2\theta}{\cos\theta}}$$

$$x = \frac{mg\sin\theta - m\omega^2 R\cos\theta}{\cos^2\theta + \sin^2\theta}$$

trig identity:

$\cos^2\theta + \sin^2\theta = 1$

$$x = mg\sin\theta - m\omega^2 R\cos\theta$$