Instructor:	Frank Secretain
Course:	Math 20
Assessment:	Final
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Marks allocated:	6 questions worth 24 marks
Percentage of final grade:	20% of final grade
Notes from instructor:	Be neat. Show your work where needed. Box final answers. Print your test and write answers in the space provided. If you can't print, then use blank paper and copy the question number as it is written on the test and answer in the space provided as if the test was printed.
Questions:	Give me a call on teams.
Submission:	At the end of your test: scan or take pictures of your test pages in order. Compile email and send it to: math20@franksecretain.ca by 10:30 am on December 18, 2020

Formula Sheet

Arithmetic Series

$$a_n = a_1 + (n-1)k$$

$$S_n = \sum_{i=1}^{n} a_1 + (i-1)k$$

$$= \frac{n}{2}(a_1 + a_n)$$

Geometric Series

$$a_n = a_1 r^{n-1}$$

$$S_n = \sum_{i=1}^{n} a_1 r^{i-1}$$

$$= a_1 \frac{1-r^n}{1-r}$$

Binomial Theorem

$$(x+y)^n = \sum_{k=0}^{n} \frac{n!}{(n-k)!k!} x^{n-k} y^k$$

Line equation

$$y = ax + b$$

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Definition of the derivative

$$\frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Rules of differentiation

$$\frac{d}{dx}(f(x)g(x)) = f(x)\frac{d}{dx}(g(x)) + g(x)\frac{d}{dx}(f(x))$$ (product rule)

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$$ (quotient rule)

$$\frac{d}{dx}(f(g(x))) = \frac{d}{dx}(f(g(x)))\frac{d}{dx}(g(x))$$ (chain rule)

Derivatives of select functions

$$\frac{d}{dx}(ax^n) = anx^{n-1}$$

$$\frac{d}{dx}(sin(x)) = cos(x)$$

$$\frac{d}{dx}(cos(x)) = -sin(x)$$

$$\frac{d}{dx}(tan(x)) = \frac{1}{(cos(x))^2}$$

$$\frac{d}{dx}(a^x) = a^x ln(a)$$

$$\frac{d}{dx}(log_a x) = \frac{1}{x ln(a)}$$

Integrals of select functions

$$\int ax^n dx = \begin{cases} \frac{a}{n+1}x^{n+1} & , n \neq -1 \\ aln(|x|) & , n = -1 \end{cases}$$ (polynomials)

$$\int sin(ax)dx = -\frac{1}{a}cos(ax)$$

$$\int cos(ax)dx = \frac{1}{a}sin(ax)$$ (trigonometry)

$$\int tan(ax)dx = \frac{1}{a}ln(|sec(ax)|)$$

$$\int a^x dx = \frac{1}{ln(a)}a^x$$

$$\int ln(x)dx = xln(x) - x$$ (exponentials)

Taylor series expansion

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_o)}{n!}(x - x_o)^n$$

Integration by parts

$$\int udv = uv - \int vdu$$

(2 marks) Determine the 100th number in the sequence and the sum from the first number to the 100th number.

2020, 1935, 1850, …

(2 marks) In infectious disease predictions, Ro is the mathematical term that indicates how contagious a disease is. Lets say we have a disease with an Ro of 4.2, therefore for every one person infected he/she with infect 4.2 other people. If you start with one person infected, how many people would be infected after just 10 steps.

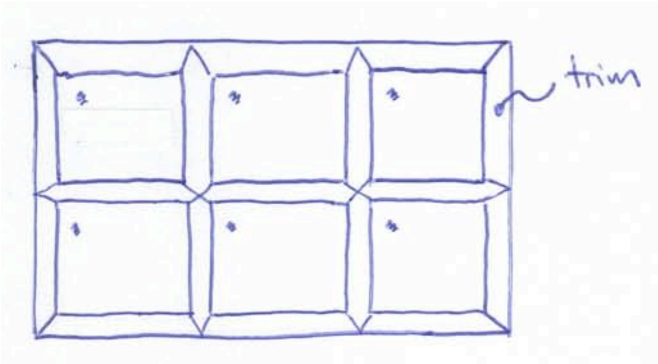
(2 marks each) Take the derivative with respect to “a” of the following functions:

$$f(a) = 4a^{\frac{3}{2}} + b\cos(ax + b)$$

$$f(a) = 4a^{\frac{3}{2}} \cos(ax + b)$$

$$f(a) = \sin(4a^{\frac{3}{2}}) + x$$

(5 marks) If you only had 12 m of trim material and wanted to build a window frame and internal grid as shown below, what would be the maximum area you could achieve:

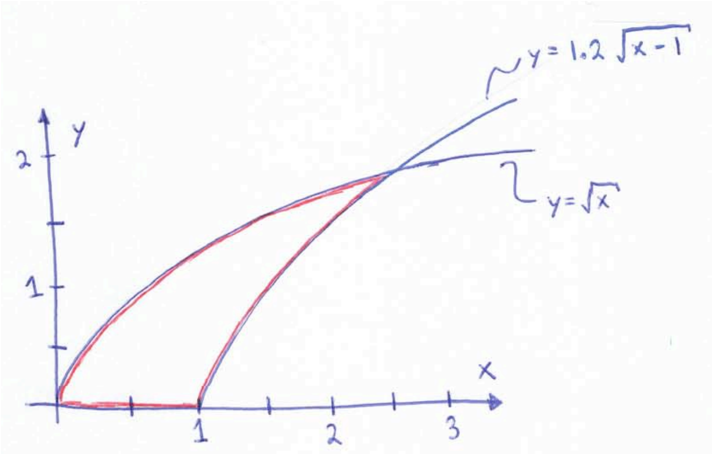


(2 marks each) Integrate with respect to "a" of the following functions:

$$\int 4a^{\frac{3}{2}} + b\cos(ax+b)\ da$$

$$\int 4a^{\frac{1}{2}}\sin(4a^{\frac{3}{2}} + b)\ da$$

(5 marks) Determine the area enclosed by the two functions and the x-axis (red outline).



(2 marks) Determine the 100th number in the sequence and the sum from the first number to the 100th number.

2020, 1935, 1850, …

$1850 - 1935 = -85$

$1935 - 2020 = -85$

$K = -85$

$n = 100$

$a_1 = 2020$

$a_n = a_1 + (n-1)K$

$= 2020 + (100-1)(-85)$

$\boxed{a_{100} = -6395}$

$S_n = \frac{n}{2}(a_1 + a_n)$

$= \frac{100}{2}(2020 + [-6395])$

$\boxed{S_{100} = -218\,750}$

(2 marks) In infectious disease predictions, Ro is the mathematical term that indicates how contagious a disease is. Lets say we have a disease with an Ro of 4.2, therefore for every one person infected he/ she with infect 4.2 other people. If you start with one person infected, how many people would be infected after just 10 steps.

$1, 4.2, 17.64, \ldots$

$r = 4.2$

$n = 10$

$a_1 = 1$

$S_n = a_1 \frac{1 - r^n}{1 - r}$

$= (1)\frac{(1 - 4.2^{10})}{(1 - 4.2)}$

$\boxed{S_{10} = 533756}$

$a_n = a_1 r^{n-1}$

$= (1)(4.2^{10-1})$

$a_{10} = 406671$

(2 marks each) Take the derivative with respect to "a" of the following functions:

$$f(a) = 4a^{\frac{3}{2}} + b\cos(ax+b)$$

$$\frac{df}{da} = 6a^{\frac{1}{2}} - bx\sin(ax+b)$$

$$f(a) = 4a^{\frac{3}{2}}\cos(ax+b)$$

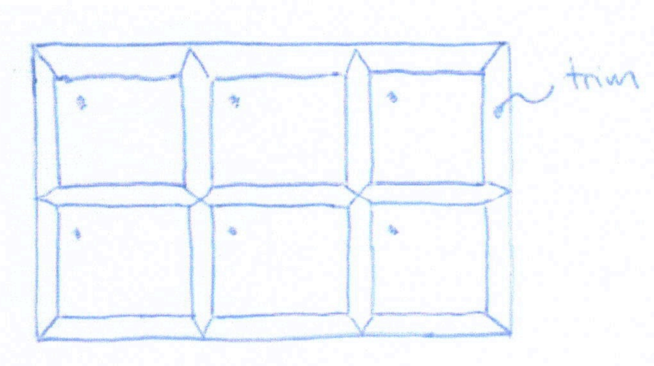
$$\frac{df}{da} = 6a^{\frac{1}{2}}\cos(ax+b) - 4a^{\frac{3}{2}}x\sin(ax+b)$$

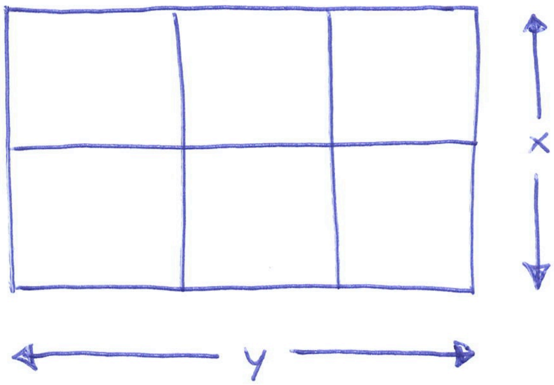
$$f(a) = \sin(4a^{\frac{3}{2}}) + x$$

$$\frac{df}{da} = \cos(4a^{\frac{3}{2}})(6a^{\frac{1}{2}})$$

$$= 6a^{\frac{1}{2}}\cos(4a^{\frac{3}{2}})$$

(5 marks) If you only had 12 m of trim material and wanted to build a window frame and internal grid as shown below, what would be the maximum area you could achieve:





$$p = 4x + 3y \qquad (1)$$

$$a = xy \qquad (2)$$

solve for x in (1)

$$x = \frac{p - 3y}{4} = \frac{12 - 3y}{4} \qquad (1a)$$

sub (1a) into (2)

$$a = \left[\frac{12 - 3y}{4}\right] y$$

$$a = 3y - \frac{3}{4}y^2$$

take derivative and set to zero

$$\frac{da}{dy} = 3 - \frac{3}{2}y = 0$$

$$y = 2$$

sub into (1a)

$$x = \frac{3}{2} = 1.5$$

So

$$\boxed{\begin{aligned} area_{max} &= (2)(1.5) \\ &= 3 \ m^2 \end{aligned}}$$

(2 marks each) Integrate with respect to "a" of the following functions:

$$\int 4a^{\frac{3}{2}} + b\cos(ax+b)\ da$$

$$\frac{8}{5} a^{\frac{5}{2}} + \frac{b}{x} \sin(ax+b) + c$$

$$\int 4a^{\frac{1}{2}} \sin(4a^{\frac{3}{2}} + b)\ da$$

let $u = 4a^{\frac{3}{2}} + b$

$\frac{du}{da} = 6a^{\frac{1}{2}}$

$da = \frac{du}{6a^{\frac{1}{2}}}$

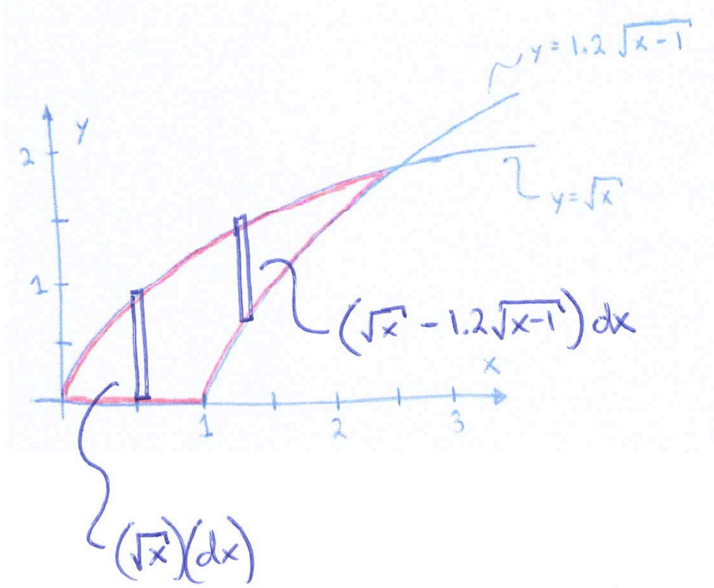
$$\int 4\cancel{a^{\frac{1}{2}}} \sin(u) \left[\frac{du}{6\cancel{a^{\frac{1}{2}}}}\right]$$

$$\frac{2}{3}\int \sin(u)\ du$$

$$-\frac{2}{3}\cos(u) + c$$

$$-\frac{2}{3}\cos(4a^{\frac{3}{2}} + b) + c$$

(5 marks) Determine the area enclosed by the two functions and the x-axis (red outline).



find inter-cept:

$$y_1 = y_2$$

$$1.2\sqrt{x-1} = \sqrt{x}$$

$$1.44(x-1) = x$$

$$1.44x - 1.44 = x$$

$$0.44x = 1.44$$

$$\boxed{x = 3.27}$$

so

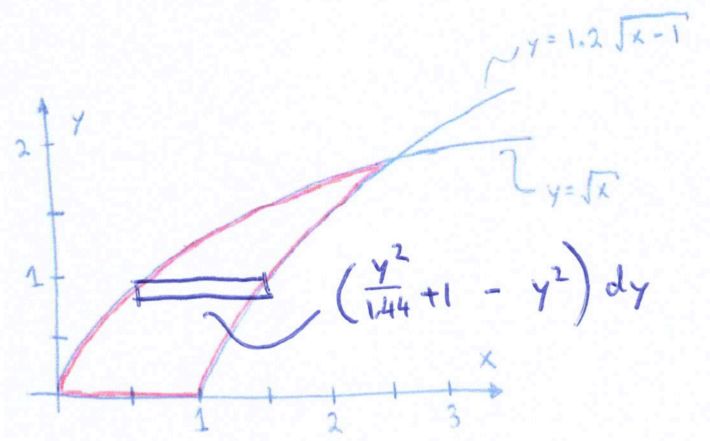
$$\int dV = \int_0^1 \sqrt{x}\,dx + \int_1^{3.27} \sqrt{x} - 1.2\sqrt{x-1}\,dx$$

$$V = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^1 + \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{2.4}{3}(x-1)^{\frac{3}{2}}\right]_1^{3.27}$$

$$= \left[\cancel{\frac{2}{3}1^{\frac{3}{2}}}\right] + \left[\frac{2}{3}(3.27)^{\frac{3}{2}} - \frac{2.4}{3}(3.27-1)^{\frac{3}{2}}\right] - \left[\cancel{\frac{2}{3}1^{\frac{3}{2}}} - \cancel{\frac{2.4}{3}(1-1)^{\frac{3}{2}}}^{\,0}\right]$$

$$\boxed{V = 1.21}$$

(5 marks) Determine the area enclosed by the two functions and the x-axis (red outline).



Find intercept:

$$y_1 = y_2$$

$$1.2\sqrt{x-1} = \sqrt{x}$$

$$1.44(x-1) = x$$

$$1.44x - 1.44 = x$$

$$0.44x = 1.44$$

$$\boxed{\begin{array}{c} x = 3.27 \\ y = \sqrt{x} = 1.81 \end{array}}$$

solve for x:

$$y = 1.2\sqrt{x-1}$$

$$x - 1 = \left(\frac{y}{1.2}\right)^2$$

$$x = \frac{y^2}{1.44} + 1$$

$$\int dV = \int_0^{1.81} \frac{y^2}{1.44} + 1 - y^2 \, dy$$

$$V = \int_0^{1.81} 1 - \frac{11}{36} y^2 \, dy$$

$$= \left[y - \frac{11}{108} y^3 \right]_0^{1.81}$$

$$= \left[1.81 - \frac{11}{108}(1.81)^3 \right] - [0]$$

$$\boxed{V = 1.21}$$