Geometric Series

Sequence of numbers such that the ratio between consective terms is a constant.

example:

$$2, 4, 8, 16, 32, \ldots$$

$$\boxed{a_n = a_1 r^{n-1}}$$

sum:

$$S_n = a_1 + a_1 r + a_1 r^2 + \ldots + a_1 r^{n-1}$$

$$rS_n = a_1 r + a_1 r^2 + \ldots + a_1 r^{n-1} + a_1 r^n$$

$$S_n - rS_n = a_1 - a_1 r^n$$

$$\boxed{S_n = \sum_{i=1}^{n} a_1 r^{i-1} = a_1 \frac{1-r^n}{1-r}, \quad r \neq 1}$$

example:

Determine the 10^{th} term of the sequence and its sum.

Q: $2, 4, 8, 16, 32, \ldots$

A: $r = \frac{32}{16} = 2$, $a_{10} = a_1 r^{n-1} = (2)(2^{10-1}) = 1024$

$$S_n = a \frac{1-r^n}{1-r} = (2)\frac{(1-2^{10})}{(1-2)} = 2046$$

Q: $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

A: $r = \frac{1}{16} \div \frac{1}{8} = \frac{1}{2}$, $a_{10} = a_1 r^{n-1} = \left(\frac{1}{2}\right)\left(\frac{1}{2}^{10-1}\right) = \frac{1}{1024}$

$$S_n = a_1 \frac{1-r^n}{1-r} = \left(\frac{1}{2}\right)\frac{(1-\frac{1}{2}^{10})}{(1-\frac{1}{2})} = \frac{1023}{1024} = 0.9990$$

Convergent / Divergent

ratio	example	infinite sum
$-1 < r < +1$	$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$	convergent
$r > +1$ or $r < -1$	$2, 4, 8, \ldots$	divergent
$r = +1$	$2, 2, 2, 2, \ldots$	divergent
$r = -1$	$2, -2, 2, -2, \ldots$	does not exist...