A fair charges an entry fee of 5$ and 1.50$ to play their game. How much will you need to play 10 games? How many games can you play for 50$?

total money spent = 1,5 , 3.0, 4.5, 6.0, ... $k = 6.0 - 4.5 = 1.5$

games = 1st game   2nd game   ...

$a_n = a_1 + (n-1)k = 1.5 + (10-1)(1.5) = 15 \Rightarrow \text{total} = \overset{\text{entry Fee}}{5\$} + \overset{\text{for 10 games}}{15\$} = 20\$$

$a_n = a_1 + (n-1)k \Rightarrow n = \frac{a_n - a_1}{k} + 1 = \frac{45 - 1.5}{1.5} + 1 = 30$

$45 = \underset{\text{\$ you have}}{50} - \underset{\text{entry}}{5}$

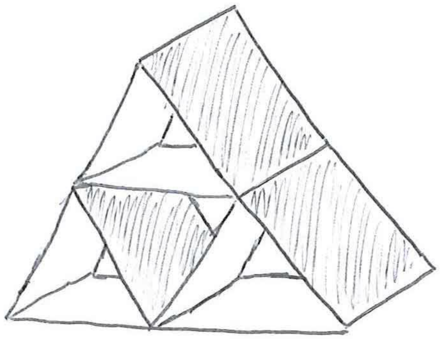
A rabbit population doubles every generation (i.e. in the 1st generation, there was 2, the second generation there was 4, and so on). How many rabbits will be in the 10th generation?

# of rabbit $= 2, 4, 8, \ldots$ $\quad r = \frac{4}{2} = 2$

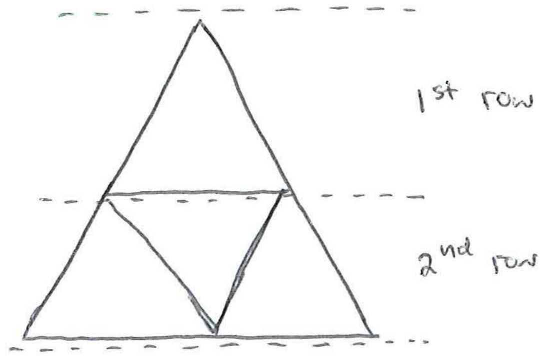
generation $= 1, 2, 3,$

$$a_n = a_1 r^{n-1} = (2)(2^{10-1}) = 1024$$

You want to make a card castle as shown in the figure below.
How many cards would be in the 10th row of the castle?
How many rows could you make with 64 cards.



isometric view



sectional view

series = 3, 6, 9, 12, ... $k = 12 - 9 = 3$

row # = 1, 2, 3, 4, ...

$$a_n = a_1 + (n-1)k = 3 + (10-1)(3) = 30$$

$$S_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}(a_1 + (a_1 + (n-1)k))$$

$$= \frac{n}{2}(2a_1 + kn - k)$$

$$= na_1 + \frac{kn^2}{2} - \frac{kn}{2}$$

$$64 = n(3) + \frac{(3)n^2}{2} - \frac{(3)n}{2}$$

$$128 = 6n + 3n^2 - 3n$$

$$0 = 3n^2 + 3n - 128$$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

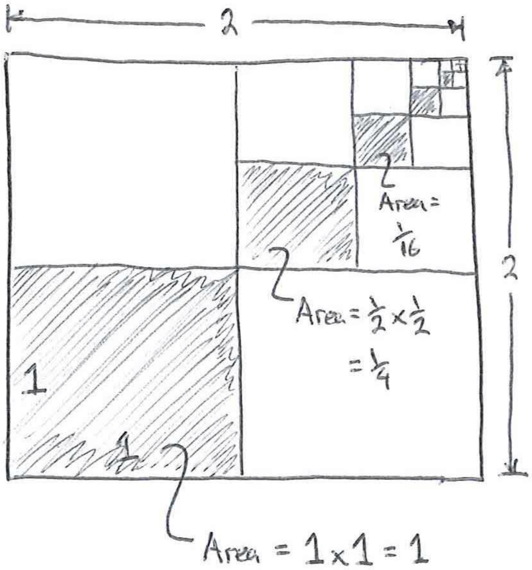
$$= \frac{-3 \pm \sqrt{3^2 - (4)(3)(-128)}}{(2)(3)}$$

$$n = \frac{-3 \pm 39.3}{6}$$

$$= 6.05 \text{ or } -7.05$$

$$n = 6$$

If a 2x2 square is halved every iteration the area is quartered (as shown in the figure). What is the area of the 10th square. What is the total area of the shaded boxes (hint: n -> infinity).



$$\text{series} = 1, \frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \ldots$$

$$\text{square \#} = 1, 2, 3, 4, \ldots$$

$$r = \frac{1/64}{1/16} = \frac{1}{4}$$

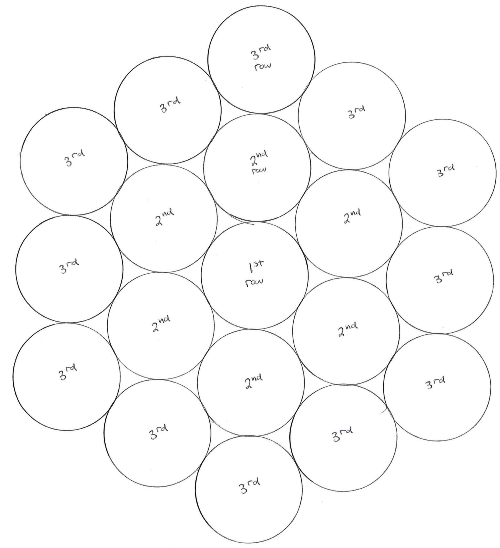
$$a_n = a_1 r^{n-1} = (1)\left(\frac{1}{4}\right)^{10-1} = \frac{1}{4^9} = \frac{1}{262144} = 3.81 \times 10^{-6}$$

$$S_n = a_1 \frac{1-r^n}{1-r} = (1)\frac{1-\cancelto{0}{\left(\frac{1}{4}\right)^{\infty}}}{1-\left(\frac{1}{4}\right)} = \frac{1}{1-\frac{1}{4}} = \frac{4}{3} = 1.\overline{3}$$

To pack wires in a single bundle one uses the hexagonal packing method shown below. The series (not counting the first row) is:

6, 12, 18, …

Determine the number of rows needed to pack 5419 wires.



arithmetic series

$$k = 18 - 12 = 6$$

$$a_n = a_1 + (n-1)k = a_1 + nk - k \qquad (1)$$

$$S_n = \frac{n}{2}(a_1 + a_n) \qquad (2)$$

sub equ. (1) into (2)

$$S_n = \frac{n}{2}\left[a_1 + (a_1 + nk - k)\right] = na_1 + \frac{k}{2}n^2 - \frac{k}{2}n$$

"not counting the first row"

$$5419 - 1 = 5418$$

rearrange

$$0 = \left(\frac{k}{2}\right)n^2 + \left(a_1 - \frac{k}{2}\right)n - S_n = \left(\frac{6}{2}\right)n^2 + \left(6 - \frac{6}{2}\right)n - 5418$$

$$= 3n^2 + 3n - 5418$$

quadratic formula

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{3^2 - (4)(3)(-5418)}}{(2)(3)} = -\frac{1}{2} \pm 42.5$$

$$= -43 \text{ or } 42 \Rightarrow n = 42 + \underbrace{1}_{\text{first row}} = 43 \text{ rows.}$$

$$\boxed{n = 43 \text{ rows}}$$

It can be argued that after driving 1 million kilometres you are concerned an expert driver. Suppose, on your first year of driving you drive 5000 km and for every year after you drive 1000 km more than the previous year (i.e. on the second year you drive 6000 km and the third 7000 km). How many years of driving must you drive to achieve 1 million kilometres (2 marks).

5000, 6000, 7000, ...

arithmetic series

$a_1 = 5000$

$K = 6000 - 5000 = 1000$

$S_n = 1\,000\,000$

$a_n = a_1 + (n-1)K \quad (1)$

$S_n = \frac{n}{2}(a_1 + a_n) \quad (2)$

sub equ. (1) into (2)

$S_n = \frac{n}{2}\left[a_1 + (a_1 + (n-1)k)\right]$

$S_n = na_1 + \frac{k}{2}n^2 - \frac{k}{2}n$

$0 = \left(\frac{k}{2}\right)n^2 + \left(a_1 - \frac{k}{2}\right)n + (-S_n)$

$0 = \left(\frac{1000}{2}\right)n^2 + \left(5000 - \frac{1000}{2}\right)n + (-1000000)$

$0 = 500n^2 + 4500n + (-1000000)$

$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-4500 \pm \sqrt{4500^2 - (4)(500)(-1000000)}}{(2)(500)}$

$= -4.5 \pm 44.9$

$= 40.4, -49.4$

$\boxed{n = 40.4 \text{ years.}}$