Determine the tangent line at x=1 for the following function. Plot the tangent line on the provided plot.

$$y = 2x^3 - x + 1$$

y value at $x = 1$

$y(x) = 2x^3 - x + 1$

$y(1) = 2(1)^3 - (1) + 1 = 2$

derivative at $x = 1$

$\frac{dy}{dx} = 6x^2 - 1$

$\frac{dy}{dx}(1) = 6(1)^2 - 1 = 5$

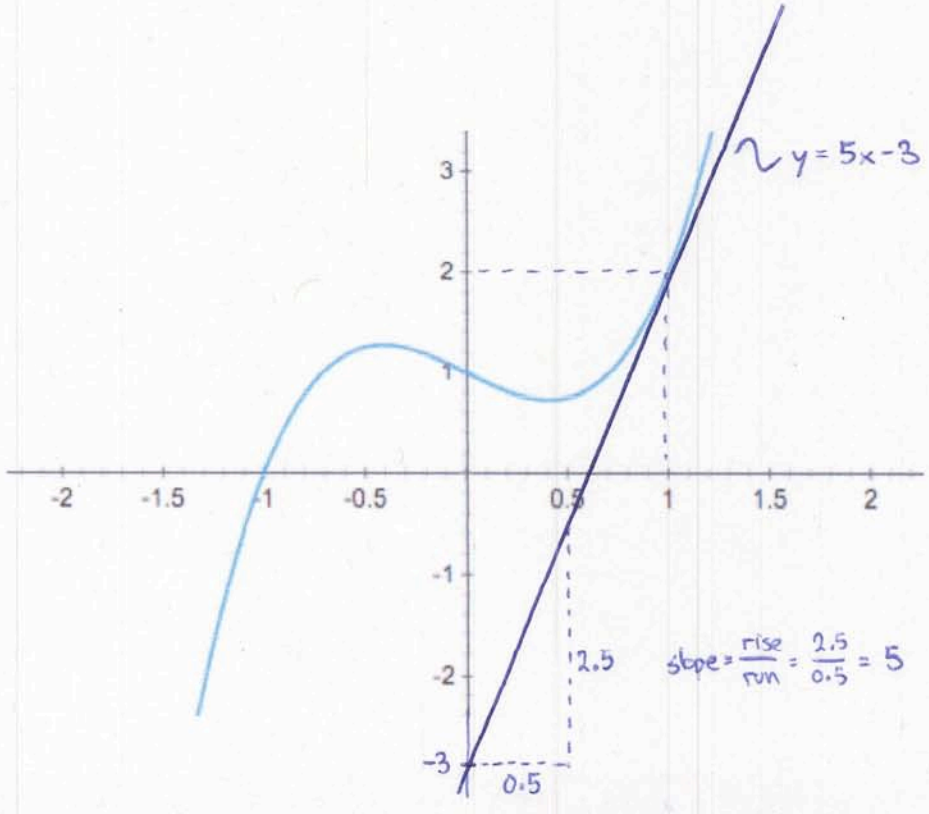
tangent line at $x = 1$

$y = ax + b$

$y = 5x + b$

$(2) = 5(1) + b \Rightarrow b = -3$

$\boxed{y = 5x - 3}$



Determine the tangent line at x=0 for the following function. Further, determine where the tangent line crosses the x-axis. Plot the tangent line on the provided plot.

$$y = -2x^2 - x + 1$$

y value at $x = 0$

$y(x) = -2x^2 - x + 1$

$y(0) = -2(0)^2 - (0) + 1 = 1$

crosses x-axis

$y = -x + 1$

$0 = -x + 1 \Rightarrow x = 1$

$\boxed{x_{cross} = 1}$

derivative at $x = 0$

$\frac{dy}{dx} = -4x - 1$

$\frac{dy}{dx}(0) = -4(0) - 1 = -1$

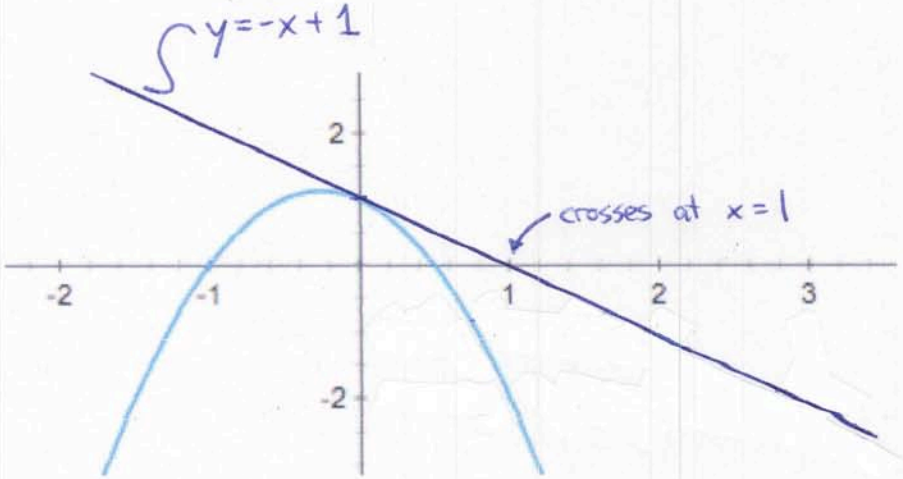
tanget line at $x = 0$

$y = ax + b$

$y = -x + b$

$(1) = -(0) + b \Rightarrow b = 1$

$\boxed{y = -x + 1}$



Determine the inflection points, concavity and tangent line at x=2 for the following function. Plot the inflection points and tangent line on the provided plot.

$$y = x^4 + 2x^3 - 5x^2 - 1$$

y value at $x = 2$

$y(x) = x^4 + 2x^3 - 5x^2 - 1$

$y(2) = (2)^4 + 2(2)^3 - 5(2)^2 - 1 = 11$

derivative at $x = 2$

$\frac{dy}{dx} = 4x^3 + 6x^2 - 10x$

$\frac{dy}{dx}(2) = 4(2)^3 + 6(2)^2 - 10(2) = 36$

tangent line at $x = 2$

$y = ax + b$

$y = 36x + b$

$(11) = 36(2) + b \Rightarrow b = -61$

$\boxed{y = 36x - 61}$

double derivative

$\frac{d^2y}{dx^2} = 12x^2 + 12x - 10$

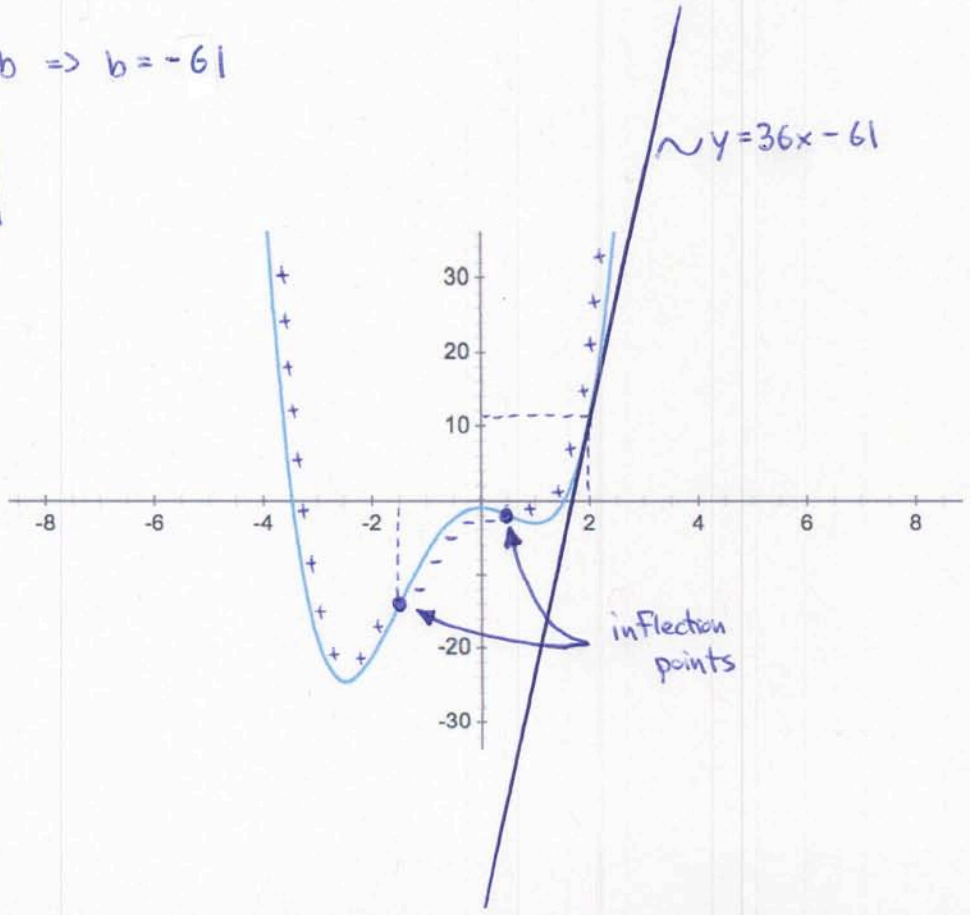
inflection points $\left(\frac{d^2y}{dx^2} = 0\right)$

$0 = 12x^2 + 12x - 10$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= -\frac{12}{24} \pm \frac{\sqrt{624}}{24}$

$\boxed{x = 0.541, -1.541}$



Determine the inflection points, concavity and tangent line at x=0.5 for the following function. Further, determine where the tangent line crosses the x-axis. Plot the inflection points and tangent line on the provided plot.

$$y = -\frac{x^{\frac{7}{3}} - 2x^{\frac{5}{3}}}{2x} + 1$$

$y = -\frac{1}{2}x^{\frac{4}{3}} + x^{\frac{2}{3}} + 1$

y value at $x = 0.5$

$y(x) = -\frac{1}{2}x^{\frac{4}{3}} + x^{\frac{2}{3}} + 1$

$y(0.5) = -\frac{1}{2}(0.5)^{\frac{4}{3}} + (0.5)^{\frac{2}{3}} + 1 = 1.43$

derivative at $x = 0.5$

$\frac{dy}{dx} = -\frac{2}{3}x^{\frac{1}{3}} + \frac{2}{3}x^{-\frac{1}{3}}$

$\frac{dy}{dx}(0.5) = -\frac{2}{3}(0.5)^{\frac{1}{3}} + \frac{2}{3}(0.5)^{-\frac{1}{3}} = 0.31$

tangent line at $x = 0.5$

$y = ax + b$

$y = 0.31x + b$

$(1.43) = (0.31)(0.5) + b \Rightarrow b = 1.27$

$\boxed{y = 0.31x + 1.27}$

crosses x-axis

$y = 0.31x + 1.27$

$0 = 0.31x + 1.27 \Rightarrow x = -4.1$

$\boxed{x_{crosses} = -4.1}$

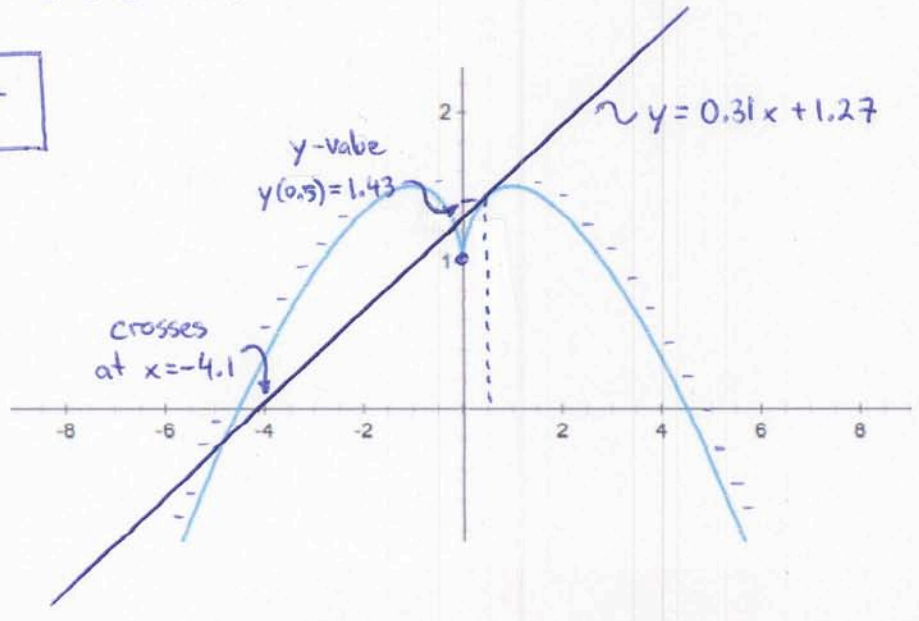
double derivative

$\frac{d^2y}{dx^2} = -\frac{2}{9}x^{-\frac{2}{3}} - \frac{2}{9}x^{-\frac{4}{3}}$

inflection point $\left(\frac{d^2y}{dx^2} = 0\right)$

$0 = -\frac{2}{9}x^{-\frac{2}{3}} - \frac{2}{9}x^{-\frac{4}{3}}$

$-x^{\frac{2}{3}} = x^{\frac{4}{3}} \Rightarrow x = 0$



Determine the tangent line that crosses the x-axis at x=2. Plot the tangent line on the provided plot.

$$y = -2x^2 + x + 1$$

derivative

$$\frac{dy}{dx} = -4x + 1$$

x_t (x value where tangent to curve)

tangent line

$$y = ax + b$$

$$\boxed{y = (-4x_t + 1)x + b}$$

at $x = 2$, $y = 0$

$$y = (-4x_t + 1)x + b$$

$$0 = (-4x_t + 1)(2) + b$$

$$\boxed{0 = -8x_t + 2 + b} \quad (1)$$

at x_t, $y_{line} = y_{Function}$

$$y_{line} = y_{Function}$$

$$(-4x_t + 1)x_t + b = -2x_t^2 + x_t + 1$$

$$-4x_t^2 + x_t + b = -2x_t^2 + x_t + 1$$

$$\boxed{0 = 2x_t^2 + 1 - b} \quad (2)$$

solve for b in (1) and sub into (2)

$$0 = 2x_t^2 + 1 - [8x_t - 2]$$

$$0 = 2x_t^2 - 8x_t + 3$$

$$x_t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

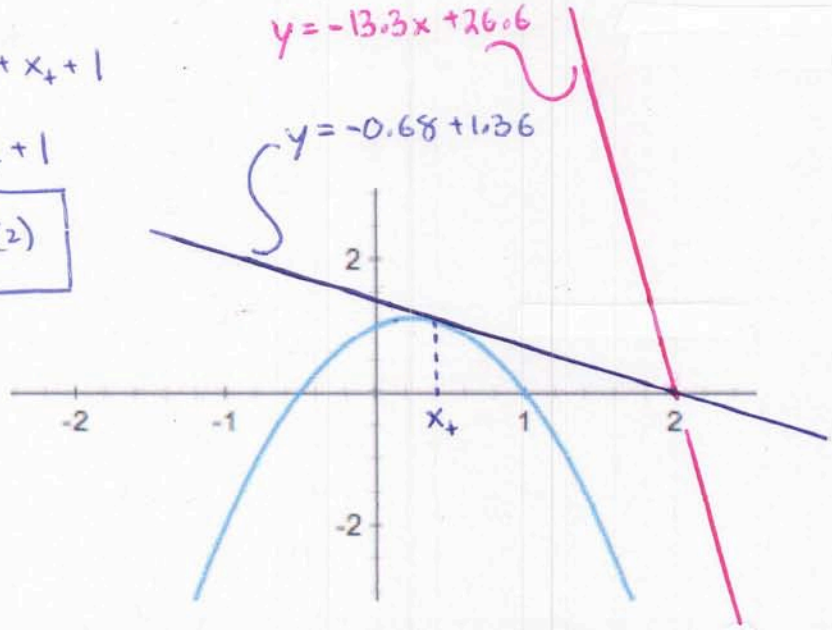
$$= 2 \pm \frac{\sqrt{40}}{4}$$

$$x_t = 3.58, 0.42$$

$$y = (-4(0.42) + 1)x + b$$

$$\boxed{y = -0.68x + 1.36}$$ solution 1

$$\boxed{y = -13.3x + 26.6}$$ solution 2



Determine the tangent line that crosses the y-axis at y=2. Plot the tangent line on the provided plot.

$$y = -2x^2 + x + 1$$

derivative

$$\frac{dy}{dx} = -4x + 1$$

(x_t value where tangent to curve)

tangent line

$$y = ax + b$$

$$\boxed{y = (-4x_t + 1)x + b}$$

at $x = 0$, $y = 2$

$$y = (-4x_t + 1)x + b$$

$$2 = (-4x_t + 1)(0) + b$$

$$\boxed{b = 2}$$

at x_t, $y_{line} = y_{function}$

$$y_{line} = y_{function}$$

$$(-4x_t + 1)x_t + 2 = -2x_t^2 + x_t + 1$$

$$-4x_t^2 + x_t + 2 = -2x_t^2 + x_t + 1$$

$$0 = 2x_t^2 - 1$$

$$\boxed{x_t = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}}}$$

$$\boxed{x_t = \pm 0.707}$$

2 solutions:

$x_t = +\frac{1}{\sqrt{2}}$, $\frac{dy}{dx} = -4\left(\frac{1}{\sqrt{2}}\right) + 1$

$= -1.83$

$x_t = -\frac{1}{\sqrt{2}}$, $\frac{dy}{dx} = -4\left(-\frac{1}{\sqrt{2}}\right) + 1$

$= 3.83$

$\boxed{y = -1.83x + 2}$ solution 1

$\boxed{y = 3.83x + 2}$ solution 2