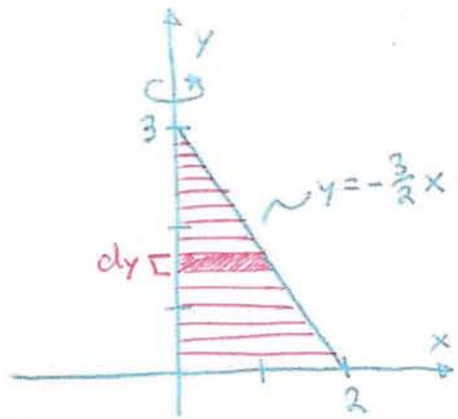
find the volume described in the figure

$x = -\frac{2}{3}y + 2$



volume of revolved triangle?

use disks of height dy and radius x(y).

$$\text{Volume} = \int_0^3 \pi r^2\, dy = \int_0^3 \pi\left(-\frac{2}{3}y + 2\right)^2 dy$$

use substitution
$u = -\frac{2}{3}y + 2$
$\frac{du}{dy} = -\frac{2}{3} \Rightarrow dy = -\frac{3du}{2}$

$$= \int \pi (u)^2 \left(-\frac{3du}{2}\right) = \int -\frac{3\pi}{2} u^2\, du$$

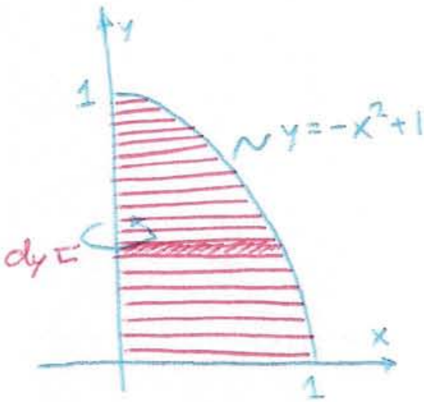
$$= \left[-\frac{3\pi}{2}\frac{1}{3}u^3\right] = \left[-\frac{\pi}{2}\left(-\frac{2}{3}y + 2\right)^3\right]_0^3$$

$$= \left[-\frac{\pi}{2}\left(-\frac{2}{3}(3) + 2\right)^3\right] - \left[-\frac{\pi}{2}\left(-\frac{2}{3}(0) + 2\right)^3\right]$$

$$= \left[0\right] - \left[-\frac{\pi}{2}(8)\right] = 4\pi \cong 12.57 \text{ units}^3$$

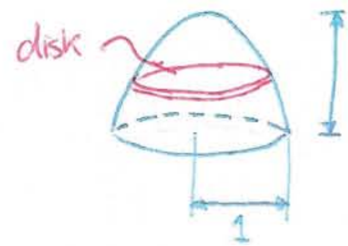
$$\boxed{\text{Volume} = 4\pi \cong 12.57 \text{ units}^3}$$

find the volume described in the figure



$x(y) = \sqrt{1-y}$

Volume of enclosed parabola



Use disks of height dy and radius x(y).

$$\text{volume} = \int_0^1 \pi r^2\,dy = \int_0^1 \pi\left(x(y)\right)^2 dy = \int_0^1 \pi\left(\sqrt{1-y}\right)^2 dy$$

$$= \int_0^1 \pi - \pi y\,dy = \left[\pi y - \frac{\pi}{2}y^2\right]_0^1$$

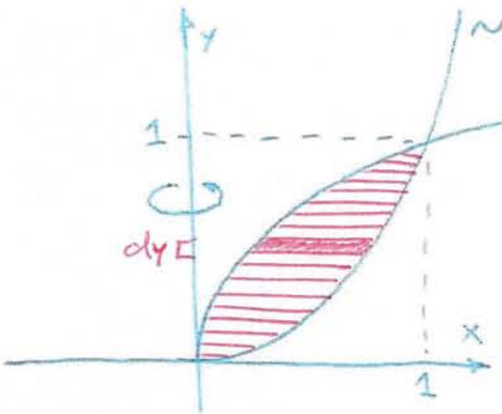
$$= \left[\pi(1) - \frac{\pi}{2}(1)^2\right] - \left[\pi(0) - \frac{\pi}{2}(0)^2\right] = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

$$\boxed{\text{volume} = \frac{\pi}{2} = 1.57 \text{ units}^3}$$

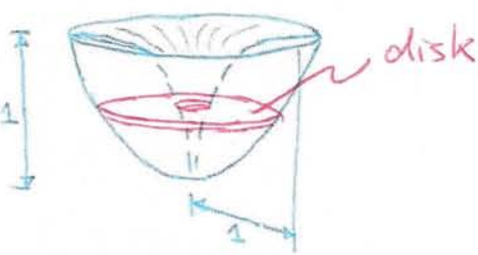
find the volume described in the figure

$x_1(y) = \sqrt{y}$

$x_2(y) = y^2$



Volume revolved by $y = \sqrt{x}$ and $y = x^2$ around the y-axis?



Use disks of height dy

and width $(\pi x_1^2 - \pi x_2^2)$

$$\text{Volume} = \int_0^1 \left(\pi x_1^2(y) - \pi x_2^2(y)\right) dy = \int_0^1 \left(\pi(\sqrt{y})^2 - \pi(y^2)^2\right) dy$$

$$= \int_0^1 \pi y - \pi y^4 \, dy = \left[\frac{\pi}{2} y^2 - \frac{\pi}{5} y^5\right]_0^1$$

$$= \left[\frac{\pi}{2}(1)^2 - \frac{\pi}{5}(1)^5\right] - \left[\frac{\pi}{2}(0)^2 - \frac{\pi}{5}(0)^5\right]$$

$$= \frac{\pi}{2} - \frac{\pi}{5} = \frac{3\pi}{10} \cong 0.942 \text{ units}^3$$

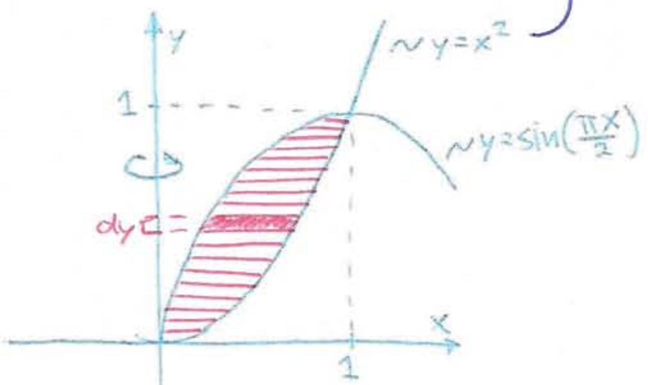
$$\boxed{\text{Volume} = \frac{3\pi}{10} \cong 0.942 \text{ units}^3}$$

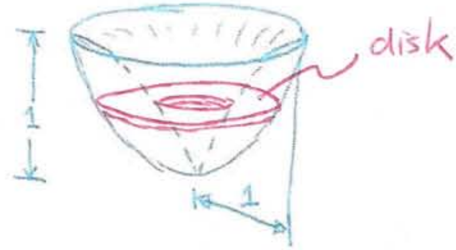
find the volume described in the figure

$x_1(y) = \sqrt{y}$

$x_2(y) = \frac{2}{\pi}\sin^{-1}(y)$

volume revolved by $y = \sin(\frac{\pi x}{2})$ and $y = x^2$ around the y-axis?





use disks of height dy and width $(\pi x_1^2 - \pi x_2^2)$

$$\text{Volume} = \int_0^1 (\pi x_1^2 - \pi x_2^2)\,dy = \int_0^1 \left(\pi(\sqrt{y})^2 - \pi\left(\tfrac{2}{\pi}\sin^{-1}(y)\right)^2\right) dy$$

$$= \int_0^1 \pi y\,dy - \int_0^1 \frac{4}{\pi}\left(\sin^{-1}(y)\right)^2 dy$$

see next page ...

$$= \left[\frac{\pi}{2}y^2\right]_0^1 - \left[\frac{4}{\pi}y\left(\sin^{-1}(y)\right)^2 + \frac{4}{\pi}\left(2\sin^{-1}(y)\right)\left(1-y^2\right)^{\frac{1}{2}} - \frac{4}{\pi}(2y)\right]_0^1$$

$$= \left(\left[\frac{\pi}{2}(1)^2\right] - \left[\frac{4}{\pi}(1)\left(\sin^{-1}(1)\right)^2 + \frac{4}{\pi}\left(2\sin^{-1}(1)\right)\left(1-(1)^2\right)^{\frac{1}{2}} - \frac{4}{\pi}(2(1))\right]\right) -$$

$$\left(\left[\frac{\pi}{2}(0)^2\right] - \left[\frac{4}{\pi}(0)\left(\sin^{-1}(0)\right)^2 + \frac{4}{\pi}\left(2\sin^{-1}(0)\right)\left(1-(0)^2\right)^{\frac{1}{2}} - \frac{4}{\pi}(2(0))\right]\right)$$

$$= \left(\left[\frac{\pi}{2}\right] - \left[\pi + 0 - \frac{8}{\pi}\right]\right) - \left(\left[0\right] - \left[0 + 0 - 0\right]\right) = \frac{8}{\pi} - \frac{\pi}{2}$$

$$\boxed{\text{Volume} = \frac{8}{\pi} - \frac{\pi}{2} \cong 0.976 \text{ units}^3}$$