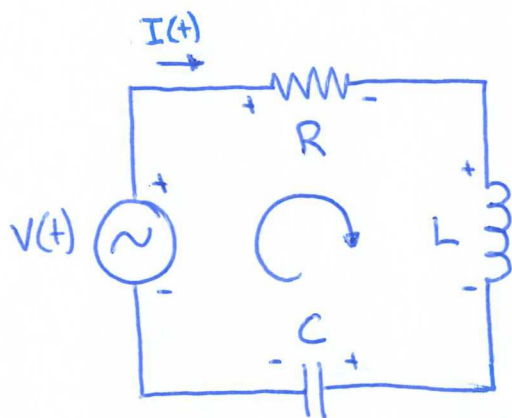


RLC Differential Equation



Kirchoff's Voltage Law

$$+V(t) - IR - L \frac{dI}{dt} - \frac{1}{C} \int I dt = 0$$

differentiate

$$L \frac{d^2 I}{dt^2} + R \frac{dI}{dt} + \frac{1}{C} I = \frac{dV}{dt}$$

(1)

solve for highest order derivative ($\frac{d^2 I}{dt^2}$) in (1)

$$\frac{d^2 I}{dt^2} = \frac{1}{L} \left[\frac{dV}{dt} - R \frac{dI}{dt} - \frac{1}{C} I \right]$$

(1a)

introduce 2 variables:

let:

$$x_1 = I$$

differentiate:

$$\frac{dx_1}{dt} = x_2$$

$$x_2 = \frac{dI}{dt}$$

$$\frac{dx_2}{dt} = \frac{1}{L} \left[\frac{dV}{dt} - R x_2 - \frac{1}{C} x_1 \right] \quad (1b)$$

setup calculation table:

equ. (1a) replacing ↑
variables with state variables (x's)

index	time	$V(t)$	$\frac{dV}{dt}$	x_1	x_2	$\frac{dx_2}{dt}$
						equ. (1b)

use: $V = V_p \sin(\omega t)$

so: $\frac{dV}{dt} = V_p \omega \cos(\omega t)$

R series L series C Differential Equation Numerical Solution

Variables

R 10 ohm
L 1 Henry = ohm s
C 0.001 Farad = s / ohm

Initial Conditions

t_o 0 s
I_o 0.9 amps <- non-zero initial current
dI/dt_o 0 amps/s

Sine Displacement (forcing function)

V_p 0 volts <- zero amplitude forcing function
frequency 10 Hz = cycles/s
omega = 62.83 rads/s (omega = 2*pi*frequency)
Period = 0.10 s (Period = 1/frequency)

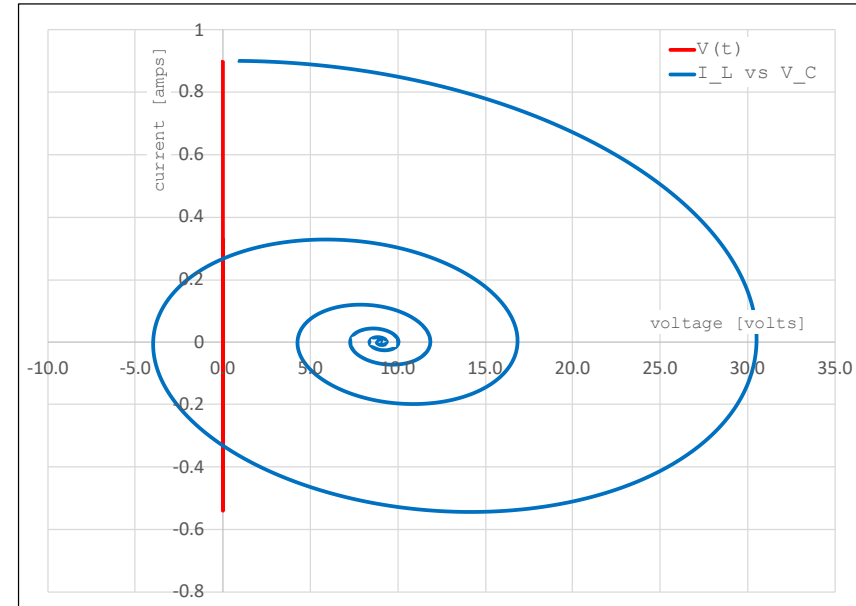
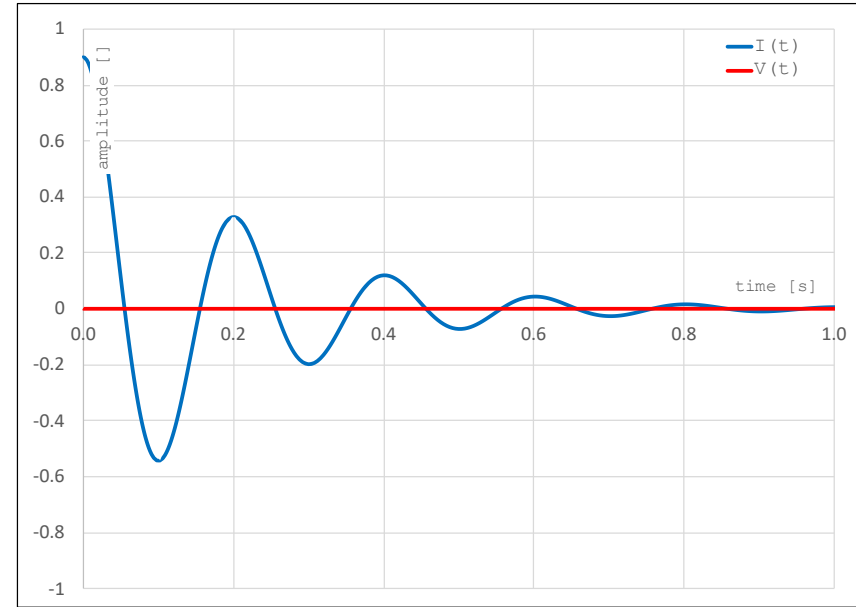
Numerical Parameters

dt = 0.0010 s

Numerical Solution

time [s]	V(t) [volts]	dV/dt [volts/s]	x_1 (I) [amps]	x_2 (dI/dt) [amps/s]	dx_2/dt (d2x/dt2) [amps/s2]	V_C [volts]	I_L [amps]
0.0000	0.0000	0.0000	0.9000	0.0000	-900.0000	0.9000	0.9000
0.0010	0.0000	0.0000	0.8991	-0.9000	-890.1000	1.7991	0.8991
0.0020	0.0000	0.0000	0.8973	-1.7901	-879.4089	2.6964	0.8973
0.0030	0.0000	0.0000	0.8946	-2.6695	-867.9453	3.5911	0.8946
0.0040	0.0000	0.0000	0.8911	-3.5375	-855.7284	4.4822	0.8911
0.0050	0.0000	0.0000	0.8867	-4.3932	-842.7779	5.3689	0.8867
0.0060	0.0000	0.0000	0.8815	-5.2360	-829.1142	6.2503	0.8815
0.0070	0.0000	0.0000	0.8754	-6.0651	-814.7580	7.1257	0.8754
0.0080	0.0000	0.0000	0.8685	-6.8798	-799.7306	7.9943	0.8685
0.0090	0.0000	0.0000	0.8608	-7.6796	-784.0537	8.8551	0.8608
0.0100	0.0000	0.0000	0.8524	-8.4636	-767.7495	9.7075	0.8524
0.0110	0.0000	0.0000	0.8432	-9.2314	-750.8407	10.5507	0.8432
0.0120	0.0000	0.0000	0.8332	-9.9822	-733.3501	11.3838	0.8332
0.0130	0.0000	0.0000	0.8225	-10.7156	-715.3010	12.2063	0.8225
0.0140	0.0000	0.0000	0.8110	-11.4309	-696.7171	13.0173	0.8110
0.0150	0.0000	0.0000	0.7989	-12.1276	-677.6224	13.8162	0.7989
0.0160	0.0000	0.0000	0.7861	-12.8052	-658.0410	14.6023	0.7861
0.0170	0.0000	0.0000	0.7726	-13.4632	-637.9973	15.3749	0.7726
0.0180	0.0000	0.0000	0.7585	-14.1012	-617.5161	16.1335	0.7585
0.0190	0.0000	0.0000	0.7438	-14.7188	-596.6222	16.8773	0.7438
0.0200	0.0000	0.0000	0.7285	-15.3154	-575.3406	17.6058	0.7285
0.0210	0.0000	0.0000	0.7126	-15.8907	-553.6965	18.3184	0.7126
0.0220	0.0000	0.0000	0.6962	-16.4444	-531.7151	19.0145	0.6962
0.0230	0.0000	0.0000	0.6792	-16.9761	-509.4218	19.6937	0.6792
0.0240	0.0000	0.0000	0.6617	-17.4855	-486.8421	20.3554	0.6617
0.0250	0.0000	0.0000	0.6437	-17.9724	-464.0012	20.9991	0.6437
0.0260	0.0000	0.0000	0.6253	-18.4364	-440.9248	21.6244	0.6253
0.0270	0.0000	0.0000	0.6064	-18.8773	-417.6383	22.2308	0.6064
0.0280	0.0000	0.0000	0.5871	-19.2950	-394.1669	22.8180	0.5871
0.0290	0.0000	0.0000	0.5674	-19.6891	-370.5361	23.3854	0.5674
0.0300	0.0000	0.0000	0.5474	-20.0597	-346.7711	23.9328	0.5474
0.0310	0.0000	0.0000	0.5270	-20.4064	-322.8970	24.4597	0.5270
0.0320	0.0000	0.0000	0.5062	-20.7293	-298.9387	24.9659	0.5062
0.0330	0.0000	0.0000	0.4852	-21.0283	-274.9210	25.4511	0.4852
0.0340	0.0000	0.0000	0.4639	-21.3032	-250.8686	25.9150	0.4639
0.0350	0.0000	0.0000	0.4423	-21.5541	-226.8059	26.3574	0.4423
0.0360	0.0000	0.0000	0.4206	-21.7809	-202.7570	26.7780	0.4206
0.0370	0.0000	0.0000	0.3986	-21.9836	-178.7458	27.1765	0.3986

simulates a
step response



R series L series C Differential Equation Numerical Solution

Variables

R 5 ohm
L 0.1 Henry = ohm s
C 0.001 Farad = s / ohm

Initial Conditions

t_o 0 s
I_o 0 amps
dI/dt_o 0 amps/s

Sine Displacement (forcing function)

V_p 0.9 volts
frequency 10 Hz = cycles/s
omega = 62.83 rads/s (omega = 2*pi*frequency)
Period = 0.10 s (Period = 1/frequency)

Numerical Parameters

dt = 0.0010 s

Numerical Solution

time [s]	V(t) [volts]	dV/dt [volts/s]	x_1 (I) [amps]	x_2 (dI/dt) [amps/s]	dx_2/dt (d2x/dt2) [amps/s2]	V_C [volts]	I_L [amps]
0.0000	0.0000	56.5487	0.0000	0.0000	565.4867	0.0000	0.0000
0.0010	0.0565	56.4371	0.0006	0.5655	530.4416	0.0006	0.0006
0.0020	0.1128	56.1028	0.0017	1.0959	489.6171	0.0022	0.0017
0.0030	0.1686	55.5470	0.0032	1.5855	443.7235	0.0055	0.0032
0.0040	0.2238	54.7721	0.0053	2.0293	393.4951	0.0108	0.0053
0.0050	0.2781	53.7810	0.0077	2.4228	339.6817	0.0184	0.0077
0.0060	0.3313	52.5776	0.0105	2.7624	283.0395	0.0289	0.0105
0.0070	0.3832	51.1668	0.0135	3.0455	224.3241	0.0424	0.0135
0.0080	0.4336	49.5540	0.0168	3.2698	164.2820	0.0592	0.0168
0.0090	0.4822	47.7456	0.0202	3.4341	103.6434	0.0794	0.0202
0.0100	0.5290	45.7488	0.0237	3.5377	43.1160	0.1032	0.0237
0.0110	0.5737	43.5715	0.0273	3.5809	-16.6217	0.1305	0.0273
0.0120	0.6161	41.2222	0.0309	3.5642	-74.9258	0.1614	0.0309
0.0130	0.6561	38.7102	0.0344	3.4893	-131.1923	0.1958	0.0344
0.0140	0.6935	36.0455	0.0377	3.3581	-184.8613	0.2335	0.0377
0.0150	0.7281	33.2385	0.0409	3.1732	-235.4208	0.2744	0.0409
0.0160	0.7599	30.3003	0.0439	2.9378	-282.4098	0.3183	0.0439
0.0170	0.7887	27.2425	0.0465	2.6554	-325.4212	0.3648	0.0465
0.0180	0.8143	24.0773	0.0488	2.3300	-364.1029	0.4136	0.0488
0.0190	0.8368	20.8170	0.0508	1.9659	-398.1597	0.4644	0.0508
0.0200	0.8560	17.4745	0.0524	1.5677	-427.3536	0.5168	0.0524
0.0210	0.8717	14.0631	0.0535	1.1404	-451.5039	0.5703	0.0535
0.0220	0.8841	10.5962	0.0542	0.6889	-470.4866	0.6245	0.0542
0.0230	0.8929	7.0874	0.0544	0.2184	-484.2336	0.6789	0.0544
0.0240	0.8982	3.5507	0.0542	-0.2658	-492.7306	0.7331	0.0542
0.0250	0.9000	0.0000	0.0534	-0.7586	-496.0155	0.7865	0.0534
0.0260	0.8982	-3.5507	0.0521	-1.2546	-494.1760	0.8386	0.0521
0.0270	0.8929	-7.0874	0.0504	-1.7488	-487.3467	0.8890	0.0504
0.0280	0.8841	-10.5962	0.0482	-2.2361	-475.7056	0.9372	0.0482
0.0290	0.8717	-14.0631	0.0454	-2.7118	-459.4713	0.9826	0.0454
0.0300	0.8560	-17.4745	0.0423	-3.1713	-438.8990	1.0249	0.0423
0.0310	0.8368	-20.8170	0.0387	-3.6102	-414.2768	1.0635	0.0387
0.0320	0.8143	-24.0773	0.0346	-4.0245	-385.9213	1.0982	0.0346
0.0330	0.7887	-27.2425	0.0302	-4.4104	-354.1741	1.1284	0.0302
0.0340	0.7599	-30.3003	0.0255	-4.7646	-319.3975	1.1539	0.0255
0.0350	0.7281	-33.2385	0.0204	-5.0840	-281.9698	1.1742	0.0204
0.0360	0.6935	-36.0455	0.0150	-5.3659	-242.2821	1.1892	0.0150
0.0370	0.6561	-38.7102	0.0094	-5.6082	-200.7334	1.1986	0.0094

