Transform n^{th} order ordinary differential equation to n 1^{st} order ordinary differential equations and numerically solve.

$$\boxed{\frac{d^n y}{dt^n} = f\left(y, \frac{dy}{dt}, \frac{d^2y}{dt^2}, \ldots, t\right)}$$

n^{th} order ODE

ordinary differential equation

introduce n variables (x) to represent each y-variable in $f(\ldots)$:

let:

$$x_1 = y$$

$$x_2 = \frac{dy}{dt}$$

$$x_3 = \frac{d^2y}{dt^2}$$

$$\vdots$$

$$x_n = \frac{d^{n-1}y}{dt^{n-1}}$$

differentiate:

$$\frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = x_3$$

$$\frac{dx_3}{dt} = x_4$$

$$\vdots$$

$$\frac{dx_n}{dt} = f(x_1, x_2, x_3, \ldots, t)$$

differentiate each equation and relate to the introduced variables (x's).

Theory

integrate:

$$\int_i^{i+1} dx_1 = \int_i^{i+1} x_2\, dt$$

$$\int_i^{i+1} dx_2 = \int_i^{i+1} x_3\, dt$$

$$\int_i^{i+1} dx_3 = \int_i^{i+1} x_4\, dt$$

$$\vdots$$

$$\int_i^{i+1} dx_n = \int_i^{i+1} f(\ldots)\, dt$$

solve:

$$x_{1\{i+1\}} = x_{1\{i\}} + x_2 \Delta t$$

$$x_{2\{i+1\}} = x_{2\{i\}} + x_3 \Delta t$$

$$x_{3\{i+1\}} = x_{3\{i\}} + x_4 \Delta t$$

$$\vdots$$

$$x_{n\{i+1\}} = x_{n\{i\}} + f(\ldots) \Delta t$$

solve for furture time step $(i+1)$ in terms of current time step (i).

where: $\boxed{\Delta t = t_{i+1} - t_i}$

Numerical (Euler method)

integrate theoretical (exact) equations over a numerical (finite) time step

$$dt \longrightarrow \Delta t$$

dt: infinitesimal time step

Δt: finite time step